PURE MATHEMATICS Unit 1
FOR CAPE® EXAMINATIONS

Multiple Choice Tests

Multiple Choice Test 1

1 A
2 B
3 A
4 A
5 A
6 A
7 D
8 A
9 B
10 C
11 A
12 C
13 A
14 C
15 D
16 D
17 D
18 B
19 B
20 B
21 A
22 D
23 C
24 B
25 C
26 B
27 C
28 A
29 B
30 C
31 B
32 C
33 B
34 C
35 A
36 D
37 B
38 C
39 C
40 C
41 A
42 B
43 B
44 A
45 D

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1 A

$$\frac{1}{(\sqrt{3}+\sqrt{2})^2} = \frac{1}{3+2\sqrt{3}\sqrt{2}+2}$$

$$= \frac{1}{5+2\sqrt{6}}$$

$$= \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}}$$

$$= \frac{5-2\sqrt{6}}{25-24}$$

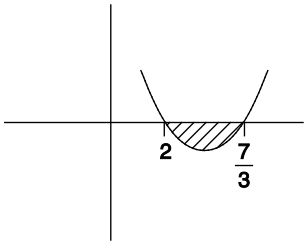
$$= 5-2\sqrt{6}$$

2 B

$3x^2 - 13x + 14 < 0$

$(3x - 7)(x - 2) < 0$

$$\left\{x: z < x < \frac{7}{3}\right\}$$



3 A

4 A

$$\sum_{r=10}^{50} r = \sum_{r=1}^{50} r - \sum_{r=1}^{9} r$$

$$= \frac{50(51)}{2} - \frac{10(9)}{2}$$

$$= 1230$$

5 A

$f(x) = 4x^3 + ax^2 + 7x + 2$

$f(2) = 0$

$$\Rightarrow \quad 4(2)^3 + a(2)^2 + 7(2) + 2 = 0$$

$$32 + 4a + 14 + 2 = 0$$

$$4a = -48$$

$$a = -12$$

6 A

$$2\log_e(5p) - 3\log_e(2p) + 2$$

$$= \log_e(5p)^2 - \log_e(2p)^3 + 2\log_e e$$

$$= \log_e\left(\frac{25p^2}{8p^3} \times e^2\right)$$

$$= \log_e\left(\frac{25e^2}{8p}\right)$$

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7 D

$P=240(1.06)^n$

$2500=240(1.06)^n$

$(1.06)^n=\frac{2500}{240}$

$n\log(1.06)=\log\left(\frac{250}{24}\right)$

$n=\frac{\log\left(\frac{250}{24}\right)}{\log(1.06)}$

$=40.22$

The year: 1841

8 A

$y = 4x - 7$

$x = 4y - 7$

$y = \frac{x + 7}{4}$

$f^{-1}(x) = \frac{x + 7}{4}$

$fg(x) = x + 1$

$g(x)=f^{-1}(x + 1)$

$=\frac{x + 1 + 7}{4}$

$=\frac{x + 8}{4}$

$\therefore g^{-1}(x) = \frac{1}{4}x + 2, x \in \square$

9 B

$y=\frac{x + 1}{x - 2}, x=\frac{y + 1}{y - 2}$

$xy - 2x = y + 1$

$xy - y = 1 + 2x$

$y=\frac{1 + 2x}{x - 1}$

$g^{-1}(4) = \frac{1 + 2(4)}{4 - 1} = \frac{9}{3} = 3$

$hg^{-1}(4) = \frac{3a + 3}{3} = 6$

$3a = 15$

$a = 5$

10 C

$x^3 - 6x^2 + 11x - 6 = 0$

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$\sum\alpha = 6$

$\sum\alpha\beta = 11$

$\alpha\beta\gamma = 6$

$\sum\alpha^2 = \left(\sum\alpha\right)^2 - 2\sum\alpha\beta$

$= (6)^2 - 2(11)$

$= 36 - 22 = 14$

11 A

$2x^3 - x^2 + 3x - 4 = 0$

$y = \frac{1}{x},\ x = \frac{1}{y}$

$\frac{2}{y^3} - \frac{1}{y^2} + \frac{3}{y} - 4 = 0$

$\Rightarrow 2 - y + 3y^2 - 4y^3 = 0$

$4y^3 - 3y^2 + y - 2 = 0$

12 C

$|2x + 1| = 3$

$2x + 1 = 3,\ 2x + 1 = -3$

$x = 1,\ z = -2C$

13 A

$\frac{16^{x+1} + 4^{2x}}{2^{x-3}\,8^{x+2}}$

$= \frac{\left(2^4\right)^{x+1} + \left(2^2\right)^{2x}}{2^{x-3}\left(2^3\right)^{x+2}}$

$= \frac{2^{4x+4} + 2^{4x}}{2^{4x+3}}$ A

$= \frac{2^{4x}\left(2^4 + 1\right)}{2^{4x} \times 2^3}$

$= \frac{17}{8}$

14 C

$|2x + 3| = 1 - 2x$

$\Rightarrow (2x + 3)^2 = (1 - 2x)^2$

$\Rightarrow 4x^2 + 12x + 9 = 1 - 4x + 4x^2$

$16x = -8$

$x = -\frac{1}{2}$

15 D

$x = 1,\ x = -2,\ x = 3$

$(x - 1)(x + 2)(x - 3) = 0$

$(x^2 + x - 2)(x - 3) = 0$

$x^3 - 3x^2 + x^2 - 3x - 2x + 6 = 0$

$x^3 - 2x^2 - 5x + 6 = 0$

$p = -2,\ q = -5,\ r = 6$

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16 D

$$\sin\theta = \frac{1}{2}$$

$$\theta = n\pi + (-1)^n\left(\frac{\pi}{6}\right) n \in \square$$

17 D

$$\cos 7\theta + \cos 4\theta = 2\cos\left(\frac{11\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

18 B

$4\cos\theta + 3\sin\theta = r\cos(\theta - \alpha)$

$= r\cos\theta\cos\alpha + r\sin\theta\sin\alpha$

$$\left.\begin{aligned} r\cos\alpha &= 4 \\ r\sin\alpha &= 3 \end{aligned}\right\} \tan\alpha = \frac{3}{4}$$

$$r = \sqrt{3^2 + 4^2} = 5$$

$$\therefore\ 4\cos\theta + 3\sin\theta = 5\cos\left(\theta - \tan^{-1}\left(\frac{3}{4}\right)\right)$$

Min value – 5

19 B

An ellipse

20 B

$4x^2 + y^2 - 8x + 4y + 6 = 0$

$4x^2 - 8x + y^2 + 4y + 6 = 0$

$4(x - 1)^2 - 4 + (y + 2)^2 - 4 + 6 = 0$

Ellipse centre (1, –2)

21 A

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}\right| \left|\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}\right|} = \frac{4}{9}$$

22 D

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} -2 \\ -2 \\ k - 2 \end{pmatrix}$$

$$\overrightarrow{AB} \cdot \overrightarrow{AB} = 0$$

$$\Rightarrow \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -2 \\ k - 2 \end{pmatrix} = 0$$

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$\Rightarrow -4 - 4 + 3k - 6 = 0$

$3k = 14$

$k = \frac{14}{3}$

23 C

$\overrightarrow{OA}\begin{pmatrix}1\\4\\2\end{pmatrix}\ \overrightarrow{OB}\begin{pmatrix}2\\5\\-1\end{pmatrix}$

$r = \begin{pmatrix}1\\4\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\-3\end{pmatrix}, \lambda \in \mathbb{R}$

24 B

$\frac{\cos(A-B)}{\cos(A+B)} = \frac{7}{3}$

$\frac{\cos A\cos B + \sin A\sin B}{\cos A\cos B - \sin A\sin B} = \frac{7}{3}$

$3\cos A\cos B + 3\sin A\sin B = 7\cos A\cos B - 7\sin A\sin B$

$10\sin A\sin B = 4\cos A\cos B$

$\frac{\sin A}{\cos B} = \frac{4}{10}$

$\tan A = \frac{2}{5}\cot B$

$5\tan A = 2\cot B$

25 C

$\cot(\theta + 45)$

$= \frac{1}{\tan(\theta + 45)}$

$= \frac{1}{\frac{\tan\theta + \tan 45}{1 - \tan\theta\tan 45}}$

$= \frac{1}{\frac{t+1}{1-t}} = \frac{1-t}{1+t}$

26 B

$\frac{\tan 2\theta}{1+\sec 2\theta} = \frac{\sin 2\theta}{1 + \frac{1}{\cos 2\theta}}$

$= \frac{\sin 2\theta}{1+\cos 2\theta}$

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$$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

27 C

$$\cos A = \frac{3}{4}$$

$\cos 4A = 2\cos^2 2A - 1$

$= 2[2\cos^2 A - 1]^2 - 1$

$$= 2\left[2\left(\frac{9}{16}\right) - 1\right]^2 - 1$$

$$= \frac{1}{32} - 1 = \frac{-31}{32}$$

28 A

$x = 2\cos t$

$y = \sin t + 1$

$$\cos t = \frac{x}{2}$$

$\sin t = y - 1$

$$\cos^2 t = \frac{x^2}{4}$$

$\sin^2 t = (y-1)^2$

$$\sin^2 t + \cos^2 t = (y-1)^2 + \frac{x^2}{4}$$

$$1 = (y-1)^2 + \frac{x^2}{4}$$

$4(y-1)^2 + x^2 = 4$

29 B

$5(x^2 + y^2) - 4x - 22y + 20 = 0$

$$x^2 + y^2 - \frac{4}{5}x - \frac{22}{5}y + 4 = 0$$

$$\text{centre } \left(\frac{2}{5}, \frac{11}{5}\right)$$

$$r = \sqrt{\frac{4}{25} + \frac{121}{25} - 4} = 1$$

30 C

$$\text{Gradient of normal} = \frac{\frac{11}{5} - \frac{8}{5}}{\frac{2}{5} - \frac{6}{5}} = \frac{\frac{3}{5}}{-4/5} = \frac{-3}{4}$$

$$\text{Gradient of the tangent} = \frac{4}{3}$$

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31 B

$$\lim_{x\to 0} \frac{\sin 3x}{2x} = \lim_{x\to 0} \frac{3}{2}\frac{\sin 3x}{3x}$$
$$= \frac{3}{2}\lim_{x\to 0}\frac{\sin 3x}{3x}$$
$$= \frac{3}{2}$$

32 C

$$\lim_{x\to 2} 4f(z) = 7 \Rightarrow \lim_{x\to 2} f(x) = \frac{7}{4}$$
$$\lim_{x\to 2}(f(x) + 2x) = \frac{7}{4} + 2(2) = \frac{23}{4}$$

33 B

$x(x + 1) = 0$
$x = 0, x = -1$

34 C

$$\frac{d}{dx}[\sin(x^3)] = 3x^2\cos(x^3)$$

35 A

$f(x) = x^2e^x$
$f'(x) = 2xe^x + x^2e^x$
$f'(0) = 0$

36 D

$P = 40$ cm
$2x + 2l = 40$
$l = 20 - x$

$$\frac{dx}{dt} = 0.5$$

$A = lx = (20 - x)\,x = 20x - x^2$

$$\frac{dA}{dx} = 20 - 2x$$

when $x = 3$, $\frac{dA}{dx} = 20 - 6 = 14$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt} = 14 \times 0.5 = 7\text{cm}^2\text{s}^{-1}$$

37 B

$2x^2 + 4xh = 150$
$4xh = 150 - 2x^2$

$$h = \frac{150 - 2x^2}{4x}$$
$$= \frac{75 - x^2}{2x}$$

38 C

$V = x^2 h$

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$$= \frac{75 - x^2}{2x} \times x^2$$

$$= \frac{75}{2}x - \frac{1}{2}x^3$$

$$\frac{dv}{dx} = \frac{75}{2} - \frac{3}{2}x^2$$

$$\frac{dv}{dx} = 0 \Rightarrow \frac{75}{2} = \frac{3}{2}x^2$$

$x^2 = 25$

$x = 5$

$$x = 5,\ V = \frac{75}{2}(5) - \frac{5^3}{2} = 125$$

39 C

$$\frac{dy}{dx} = 8 - \frac{1}{x^3}$$

$$\frac{dy}{dx} = 0 \Rightarrow 8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

when $x = \frac{1}{2},\ y = 4 + 2 = 6$

40 C

$$\lim_{x \to 0}\left(\frac{\cos 4x - 1}{x}\right)$$

$$= \lim_{x \to 0} 4\left[\frac{\cos 4x - 1}{4x}\right]$$

$= 4(0) = 0$

41 A

$$\int 4\cos 6\theta \cos 2\theta \, d\theta$$

$2\cos 6\theta \cos 2\theta = \cos 8\theta + \cos 4\theta$

$$\int 4\cos 6\theta \cos 2\theta \ d\theta$$

$$= 2\int \cos 8\theta \cos 4\theta \ d\theta$$

$$= \frac{2}{8}\sin 8\theta + \frac{2}{4}\sin 4\theta + c$$

$$= \frac{1}{4}\sin 8\theta + \frac{1}{2}\sin 4\theta + c$$

42 C

$$\int_0^{\pi/2} \tan^2(2x)\,dx = \int_0^{\pi/2} \sec^2(2x) - 1\,dx$$

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$$= \left[\frac{1\pi}{2}\tan(2x) - x\right]_0^{\pi/2} = -\frac{}{2}$$

43 B

$2x^2 + 5 = 3x^2 + 1$

$x^2 = 4$

$x = \pm 2$

$x = 12, y = 13$

$$\text{shaded area} = \int_0^2 (2x^2 + 5)dx - \int_0^2 (3x^2 + 1)dx$$

$$= \int_0^2 (-x^2 + 4)dx$$

$$= \left[-\frac{1}{3}x^3 + 4x\right]_0^2$$

$$= \frac{-8}{3} + 8 = \frac{16}{3}$$

44 A

$$\pi\int x^2 dy = \pi\int_1^{13}\left(\frac{y-1}{5}\right)dy - \pi\int_5^{13}\left(\frac{y-5}{2}\right)dy$$

$$= \left[\frac{\pi}{3}\frac{(y-1)^2}{2}\right]_1^{13} - \left[\frac{\pi}{2}\frac{(y-5)^2}{2}\right]_5^{13}$$

$$= \frac{\pi}{6}[144] - \frac{\pi}{4}[64]$$

$= 24\pi - 16\pi = 8\pi = 25.1$

45 D

$$\int_2^5 f(x)dx + [4x]_2^4$$

$= 12 + 16 - 8 = 20$

Multiple Choice Test 2

1 B
2 **C**
3 A
4 B
5 C
6 B
7 A
8 D
9 C
10 B
11 D
12 A
13 C
14 C
15 B
16 D
17 A

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18 B
19 A
20 C
21 C
22 A
23 B
24 B
25 A
26 A
27 B
28 A
29 B
30 A
31 B
32 A
33 C
34 C
35 B
36 A
37 B
38 B
39 C
40 A
41 B
42 A
43 A
44 C
45 A

1 B

$|x^2 - 3| < 1$

$\Rightarrow -1 < x^2 - 3 < 1$

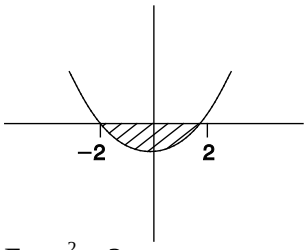
$2 < x^2 < 4$

For $x^2 < 4$

$x^2 - 4 < 0$

$(x - 2)(x + 2) < 0$

$\therefore -2 < x < 2$



For $x^2 > 2$

$x^2 - 2 > 0$

$\left(x - \sqrt{2}\right)\left(x - \sqrt{2}\right) > 0$

$x < -\sqrt{2},\ x > \sqrt{2}$

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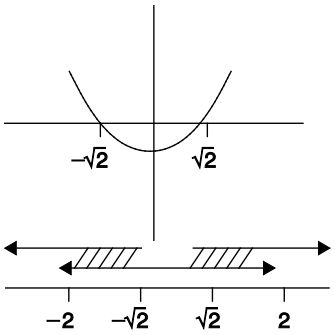

$\therefore \{x : 2 < x < -\sqrt{2}\} \cup \{x : \sqrt{2} < x < 2\}$

2 C

$$\log_2\left(\frac{\sqrt{2}}{8}\right) = \log_2 2^{\frac{1}{2}-3} = \frac{-5}{2}\log_2 2 = \frac{-5}{2}$$

3 A

$f(x) = x^3 - 7x^2 + kx - 12$
$f(2) = 2^3 - 7(2)^2 + 2k - 12 = 0$
$2k = 32,\ k = 16$

4 B

"If presentation college chaguanas do not win, then it is not raining."

5 C

Truth table:

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	$p \cap (\sim q)$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

6 B

$$f(x) = \frac{2}{3+x}$$

$$f\left(\frac{2}{3+x}\right) = \frac{2}{3 + \frac{2}{3+x}}$$

$$= \frac{2(3+x)}{11+3x}$$

$\therefore$ ff(x) is undefined at x = –3, $x = \frac{-11}{3}$

7 A

$|x + 1| = 3|2x + 1|$
$(x + 1)^2 = 3^2 (2x + 1)^2$
$x^2 + 2x + 1 = 9 (4x^2 + 4x + 1)$
$35x^2 + 34x + 8 = 0.$
$(5x + 2) (7x + 4) = 0$

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$$x = \frac{-2}{5}, -\frac{-4}{7}$$

8 D

$$\log_3\left(\frac{1}{27}\right) = \log_3 3^{-3} = -3$$

9 C

$(x - 2)(x + 3)(4x - 3) = 0$
$(x^2 + x - 6)(4x - 3) = 0$
$4x^3 - 3x^2 + 4x^2 - 3x - 24x + 18 = 0$
$4x^3 + x^2 - 27x + 18 = 0$

10 B

$$\frac{x}{x-2} < 0$$

	x	x – 2	$\frac{x}{x-2}$
x < 0	–ve	–ve	+ve
0 < x < 2	+ve	–ve	–ve
x > 2	+ve	+ve	+ve

$\therefore \{x : 0 < x < 2\}$

11 D

$$\frac{1}{\sqrt{x+2}-2} = \frac{1}{\sqrt{x+2}-2} \times \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$$
$$= \frac{\sqrt{x+2}+2}{x+2-4} = \frac{\sqrt{x+2}+2}{x-2}$$

12 A

$$(3x)^{3/2} + \left(\frac{8}{3}x\right)^{2/3} = 31$$
$$x = 3, \quad 9^{3/2} + 8^{2/3} = 27 + 4 = 31$$

13 C

$x^3 + 6x^2 + 11x + 6 = 0$
$(-1)^3 + 6(-1)^2 + 11(-1) + 6 = 0.$
$x^3 + 6x^2 + 11x + 6 = (x + 1)(x^2 + 5x + 6)$
$= (x + 1)(x + 2)(x + 3)$
$x = -1, -2, -3$

14 C

$x^3 - 2x^2 + 4x - 7 = 0$
$\sum\alpha = 2$
$\sum\alpha\beta = 4$
$\alpha\beta\gamma = 7$
$\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$
$= (2)^2 - 2(4)$
$= -4$

15 B

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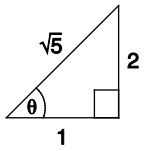
$a * b = 2ab + a - 3$
$a * e = a$
$\Rightarrow 2ae + a - 3 = a$
$e = \frac{3}{2a}$

16 D
$\sin 9\theta - \sin 3\theta$
$= 2\cos 6\theta \sin 3\theta$

17 A
$\tan \theta = 2$
$\sin\theta = \frac{2}{\sqrt{5}}$



18 B
$f(x) = 1 - 6 \sin x \cos x + 4\cos^2 x$
$= 1 - 3\sin 2x + 2[\cos 2x + 1], \quad \cos 2x = 2\cos^2 x - 1$
$= 3 - [3\sin 2x - 2\cos 2x]$
$3 \sin 2x - 2 \cos 2x = R \sin (2x - \alpha)$
$= R \sin 2x \cos \alpha - R \cos 2x \sin \alpha$
$R \cos \alpha = 3$
$R \sin \alpha = 2$
$\tan \alpha = 2/3 \Rightarrow \alpha = 33.7°$
$R = \sqrt{3^2 + (-2)^2} = \sqrt{13}$
$\therefore f(x) = 3 - \sqrt{13}\sin(2x - 33.7°)$

19 A
$3 + \sqrt{13}$ when $\sin (2x - 33.7°) = -1$

20 C
$\sin (2x - 33.7°) = -1$
$2x = -90 + 33.7°, 270 + 33.7°$
$x = 151.9°$

21 C
$$\overrightarrow{OA} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} -3 \\ m+2 \\ -1 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} -2 \\ 4 \\ -5 \end{pmatrix}$$
$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ m \\ 0 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 1 \\ 2-m \\ -4 \end{pmatrix}$$
$$\overrightarrow{AB}.\overrightarrow{BC} = 0 \Rightarrow \begin{pmatrix} -1 \\ m \\ 0 \end{pmatrix}.\begin{pmatrix} 1 \\ 2-m \\ -4 \end{pmatrix} = 0$$
$-1 + (2 - m)\, m = 0$

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$m^2 - 2m + 1 = 0$
$(m-1)^2 = 0 \Rightarrow m = 1$

22 A

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$$

$$\overrightarrow{AB} . \overrightarrow{AC} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} . \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} = 5$$

23 B

$$\cos\theta = \frac{5}{\sqrt{9}\sqrt{10}}$$

$\theta = 58.2°$

24 B

$$r = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$$

$$\frac{x-1}{2} = \frac{y-2}{3} = z - 4$$

25 A

$$r . \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} . \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$= 10$
$4x + 2y = 10$
$2x + y = 5$

26 A

$x^2 + 4x + y^2 - y + 2 = 0$
$(x+2)^2 - 4 + (y-1)^2 - 1 + 2 = 0$

$$(x+2)^2 + (y-1)^2 = (\sqrt{3})^2$$

$$r = \sqrt{3}$$

27 B

$x = 2 \sec t$

$$\sec^2 t = \frac{x^2}{4}$$

$\tan t = y - 1$
$\tan^2 t = (y-1)^2$

$$1 = \frac{x^2}{4} - (y-1)^2$$

$$(y-1)^2 = \frac{x^2}{4} - 1$$

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$$y - 1 = \sqrt{\frac{x^2 - 4}{4}}$$

$$y = 1 + \frac{\sqrt{x^2 - 4}}{2}$$

$$= \frac{2 + \sqrt{x^2 - 4}}{2}$$

28 A

$\cos^2 \theta = (x - 2)^2$

$\sin^2 \theta = (y - 3)^2$

$(y - 3)^2 + (x - 2)^2 = 1$

Circle centre (2, 3)

29 B

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}, \lambda \in \mathbb{R}$$

$$\left.\begin{aligned} x &= 2 + 4\lambda \\ y &= 1 + \lambda \\ z &= -4 + 5\lambda \end{aligned}\right\} \lambda \in \mathbb{R}$$

30 A

$x^2 + y^2 = 49$

Circle centre (0, 0), r = 7

$x^2 - 6x + y^2 - 8y + 21 = 0$

$(x - 3)^2 - 9 + (y - 4)^2 - 16 + 21 = 0$

$(x - 3)^2 + (y - 4)^2 = 4$ A

Circle centre (3, 4), r = 2

31 B

$$\lim_{x \to 0}\left(\frac{\sin 4x}{x}\right) = 4\lim_{x \to 0}\left(\frac{\sin 4x}{4x}\right) = 4$$

32 A

$$\lim_{x \to -2} \frac{4x^2 + 10x + 4}{3x + 6}$$

$$= \lim_{x \to -2} \frac{2(2x + 1)\cancel{(x + 2)}}{3\cancel{(x + 2)}}$$

$$= \frac{2}{3}(-4 + 1) = -2$$

33 C

$x^2 - 5x + 5 = 0$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

= 3.62, 1.38

34 C

$x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$x(x - 2)(x - 1) = 0$

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x = 0 , 1 , 2

35 B

$$\lim_{x\to 0}\left(\frac{x^2-3x}{2}\right)=\frac{0-0}{2}=0$$

36 A

f(x) = x cos 3x

f ′(x) = cos 3x – 3x sin 3x

37 B

$y = ax^2 + bx - 3$

$$\frac{dy}{dx} = 2ax + b$$

$$x = -4, \frac{dy}{dx} = 9 \Rightarrow -8a + b = 9 \qquad [1]$$

$x = -4, y = -31 \Rightarrow -31 = 16a - 4b - 3$

$16a - 4b = -28$ [2]

$2 \times [1] + [2] \Rightarrow -2b = 10$

$b = 5$

$$a = \frac{-1}{2}$$

38 B

$$y = 2x^3 - 9x^2 + 12x$$

$$\frac{dy}{dx} = 6x^2 - 18x + 12 = 0$$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1, 2$

$x = 1, y = 2 - 9 + 12 = 5$

$x = 2, y = 16 - 36 + 24 = 4$

(1, 5) (2, 4)

39 C

$$f(x) = 4(2x + 1)^{-3}$$

$$= -24(2x + 1)^{-4}$$

$$= \frac{-24}{(2x + 1)^4}$$

40 A

$f(x) = 2x^2 + 4x - 3$

$f'(x) = 4x + 4 < 0$

$x < -1$

41 B

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x - 1) = 0$

$x = 0, x = 1$

$$\int_0^1 x^2 dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$$

$$\int_0^1 2x - x^2 dx = \left[x^2 - \frac{1}{3}x^3\right]_0^1 = \frac{2}{3}$$

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$\therefore$ Area $= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

42 A

$$\frac{dy}{dx} = 2x + 1$$

$$y = \int (2x + 1)dx$$

$$y = \frac{1}{4}(2x+1)^2 + C$$

$$x = 0,\ y = 3 \Rightarrow C = 2\frac{3}{4}$$

$$y = \frac{1}{4}(2x+1)^2 + 2\frac{3}{4}$$

43 A

$$\int_0^1 x^2(5 - \sqrt{x})$$

$$= \int_0^1 5x^2 - x^{5/2}dx$$

$$= \left[\frac{5}{3}x^3 - \frac{2}{7}x^{7/2}\right]_0^1$$

$$= \frac{5}{3} - \frac{2}{7} = \frac{29}{21}$$

44 C

$$V = \pi\int y^2 dx$$

$$V = \pi\int_0^2 (5x + 1)dx$$

$$= \pi\left[\frac{5x^2}{2} + x\right]_0^2$$

$$= \pi\left[10 + 2\right] = 12\pi$$

45 A

$$\frac{dp}{dt} \propto P$$

$$\Rightarrow \frac{dp}{dt} = KP, \quad k > 0$$

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Multiple Choice Test 3

1 B
2 C
3 C
4 A
5 B
6 A
7 D
8 C
9 A
10 B
11 D
12 B
13 A
14 C
15 C
16 D
17 A
18 D
19 B
20 B
21 C
22 A
23 C
24 B
25 D
26 A
27 B
28 C
29 A
30 D
31 B
32 C
33 B
34 D
35 A
36 D
37 B
38 A
39 D
40 C
41 D
42 B
43 C
44 C
45 D

1 B
$f(x) = 4x - 2$
$f(2) = 4(2) - 2 = 6$
$f^2(2) = f(6) = 4(6) - 2 = 22$

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2 C

3 C

p ⇔ q is logically equivalent to
~ p ⇔ ~q

4 A

$$\frac{x+3}{2x-1} > 0$$

	$x+3$	$2x-1$	$\frac{x+3}{2x-1}$
$x < -3$	−ve	−ve	+ve
$-3 < x < \frac{1}{2}$	+ve	−ve	−ve
$x > \frac{1}{2}$	+ve	+ve	+ve

$$\therefore \quad \{x : x < -3\} \cup \left\{x : x > \frac{1}{2}\right\}$$

5 B

a*e = a + e – 2ae = a
e(1– 2a) = 0
e = 0

6 A

$$\sum_{r=1}^{20}(3r+2) = 3\sum_{r=1}^{20} r + \sum_{r=1}^{20} 2$$

$$= \frac{3(20)(21)}{2} + 2(20)$$

$$= 670$$

7 D

$$\log\sqrt{(x-3)(x+3)} + \log\sqrt{\frac{x+3}{x-3}}$$

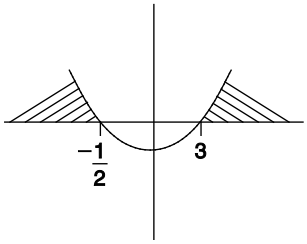
$$= \log\left(\cancel{\sqrt{x-3}}\sqrt{x+3}\right) \times \frac{\sqrt{x+3}}{\cancel{\sqrt{x-3}}}$$

$$= \log(x+3)$$

8 C

$2x^2 - 5x - 3 \geq 0$
$(2x + 1)(x - 3) \geq 0$

$$\{x : x \leq -\frac{1}{2}\} \cup \{x : x \geq 3\}$$



9 A

10 B

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$y = \frac{1}{x}, \quad x = \frac{1}{y}$

$\frac{1}{y^3} - \frac{4}{y^2} + \frac{6}{y} - 8 = 0$

$1 - 4y + 6y^2 - 8y^3 = 0$

$8y^3 - 6y^2 + 4y - 1 = 0$

11 D

$6x^3 - 3x^2 - 3x + 2 = 0$

$\sum \alpha = \frac{1}{2}$

$\sum \alpha\beta = -\frac{1}{2}$

$\sum \alpha^2 = \left(\sum \alpha\right)^2 - 2\sum \alpha\beta$

$= \left(\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right)$

$= \frac{1}{4} + 1 = \frac{5}{4}$

12 B

$|2x - 1| > 5$

$2x - 1 > 5, \quad 2x - 1 < -5$

$x > 3, \quad x < -2$

$\{x: x < -2\} \cup \{x: x > 3\}$

13 A

$f(x) = \frac{4}{x} - 3$

$y = \frac{4}{x} - 3$

$y + 3 = \frac{4}{x} \Rightarrow x = \frac{4}{y+3}$

$y = 1, x = 1,$

$y = -1 x = 2$

$y = -\frac{11}{5}, \quad x = 5$

$y = -\frac{5}{2}, \quad x = 8$

14 C

$\sqrt{2} + \sqrt{36} + \sqrt{72}$

$= \sqrt{2} + 6 + 6\sqrt{2}$

$= 6 + 7\sqrt{2}$

15 C

$f(x) = x^3 - ax^2 + 2x + 5$

$f(2) = 8 - 4a + 4 + 5 = 0$

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$4a=17$

$a=\frac{17}{4}$

16 D

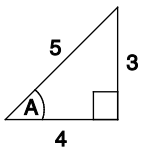
$$\frac{1-\cos 2\theta}{\sin 2\theta}=\frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

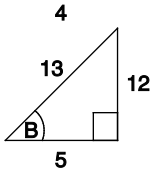
$$=\frac{\sin\theta}{\cos\theta}=\tan\theta$$

17 A

$$\cos A=\frac{4}{5}$$

$$\cos B=\frac{5}{13}$$





$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$=\left(\frac{4}{5}\right)\left(\frac{5}{13}\right)-\left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$

$$=\frac{20}{65}-\frac{36}{65}=\frac{-16}{65}$$

18 D

$$3\cos\theta+4\sin\theta=r\cos\theta\cos\alpha+r\sin\theta\sin\alpha$$

$$r\sin\alpha=4$$

$$r\cos\alpha=3$$

$$\tan\alpha=\frac{4}{3},\alpha=\arctan\left(\frac{4}{3}\right)$$

$$r=\sqrt{3^2+4^2}=5$$

$$\therefore 3\cos\theta+4\sin\theta=5\cos\left(\theta-\arctan\left(\frac{4}{3}\right)\right)$$

19 B

Max = 9, Min = 6 – 3 = 3

20 B

$$\cos\theta=\frac{x-2}{3}\Rightarrow\cos^2\theta=\frac{(x-2)^2}{9}$$

$$\sin\theta=\frac{y-3}{2}\Rightarrow\sin^2\theta=\frac{(y-3)^2}{4}$$

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$$\therefore \frac{(x-2)^2}{9}+\frac{(y-3)^2}{4}=1$$

$$\Rightarrow 4(x^2-4x+4)+9(y^2-6y+9)=36$$

$$4x^2+9y^2-16x-54y+61=0$$

21 C

22 A

$x^2 - 2x + y^2 - 4y - 4 = 0.$

$(x-1)^2 - 1 + (y-2)^2 - 4 - 4 = 0$

$(x-1)^2 + (y-2)^2 = 9$

Circle centre (1, 2), r = 3

23 C

$\cos^4 x - \sin^4 x$

$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$= \cos 2x$

24 B

$\cos 6\theta + \cos 4\theta$

$= 2\cos 5\theta \cos \theta$

25 D

$$a \cdot b = 0$$

$$\begin{pmatrix}1\\4\\5\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\P+3\end{pmatrix}=0$$

$$\Rightarrow -2+4+5P+15=0$$

$$5P=-17$$

$$P=\frac{-17}{5}$$

26 A

Mid point of $\overrightarrow{PQ}$

$$=\begin{pmatrix}4\\9\\-3\end{pmatrix}=4i+9j-3k$$

27 B

$$r\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=12$$

$$r\cdot\begin{pmatrix}1/3\\2/3\\2/3\end{pmatrix}=\frac{12}{3}=4$$

Distance = 4

28 C

$$x^2-4x+y^2-2y+1=0$$

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centre $(2,1)$

$\frac{4+a}{2}=2 \Rightarrow a=0$

$\frac{b+1}{2}=1, b=1$

$(0,1)$

29 A

$\cos 2\theta = \frac{1}{2}$

$2\theta = 2n(180) \pm 60°, \quad n \in \mathbb{Z}$

$\theta = 180°n \pm 30°, \quad n \in \mathbb{Z}$

30 D

$\cos 75 \cos 15 - \sin 75 \sin 15$

$= \cos (75 + 15) = \cos 90 = 0$

31 B

$\lim_{x\to 0} \frac{\sin 6x}{x}$

$= 6 \lim_{x\to 0} \frac{\sin 6x}{6x}$

$= 6$

32 C

$\lim_{x\to 3} \frac{x^2 - x - 6}{2x^2 - 5x - 3}$

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+2)}{(2x+1)\cancel{(x-3)}}$

$= \frac{3+2}{6+1} = \frac{5}{7}$

33 B

$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

$= \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \times \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$= \lim_{x\to 3} \frac{(x+1)-4}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2}$

$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$

34 D

$\int \frac{x^7}{x^4} - \frac{2}{x^4} dx$

$= \int (x^3 - 2x^{-4}) dx$

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$= \frac{1}{4}x^4 + \frac{2}{3x^3} + c$

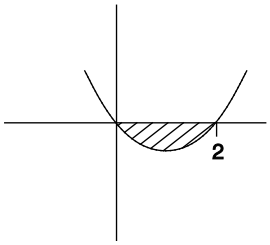
35 A

$f(x) = x^3 - 3x^2 + 2$

$f'(x) = 3x^2 - 6x < 0$

$x(x - 2) < 0$

$0 < x < 2$



36 D

$f(x) = \cos^3 x$

$f'(x) = -3\cos^2 x \sin x$

37 B

$\int 2\sin 8\theta \cos 4\theta \, d\theta$

$= \int (\sin 12\theta + \sin 4\theta) \, d\theta$

$= -\frac{1}{12}\cos 12\theta - \frac{1}{4}\cos 4\theta + C$

38 A

$|x|^2 - 4 = 0$

$x^2 = 4$

$x = 2, -2$

39 D

$V = \pi\int y^2 dx = \pi\int_0^2 (x^2+1)^2 dx$

$= \pi\int_0^2 (x^4 + 2x^2 + 1)dx = \pi\left[\frac{1}{5}x^5 + \frac{2}{3}x^3 + x\right]_0^2$

$= \pi\left[\frac{32}{5} + \frac{16}{3} + 2\right] = \frac{206}{15}\pi$

40 C

$\lim_{x\to 0} \frac{20\cos 10x}{\cos 6x \sin 6x}$

$= \lim_{x\to 0} \frac{20\sin 10x}{\frac{1}{2}\sin 12x}$

$= 40\lim_{x\to 0} \frac{10\frac{\sin 10x}{10x}}{12\frac{\sin 12x}{12x}}$

$= \frac{400}{12} = \frac{100}{3}$

41 D

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$\varphi(x) = \sec^4\left(x + \frac{\pi}{4}\right)$

$\varphi'(x) = 4\sec^3\left(x + \frac{\pi}{4}\right)\sec\left(x + \frac{\pi}{4}\right)\tan\left(x + \frac{\pi}{4}\right)$

$= 4\sec^4\left(x + \frac{\pi}{4}\right)\tan\left(x + \frac{\pi}{4}\right)$

$\varphi^2(x) = 4\sec^4\left(x + \frac{\pi}{4}\right)\sec^2\left(x + \frac{\pi}{4}\right) + 16\sec^3\left(x + \frac{\pi}{4}\right)\sec\left(x + \frac{\pi}{4}\right)\left(\tan^2\left(\frac{\pi}{4} + x\right)\right)$

$= 4\sec^6\left(x + \frac{\pi}{4}\right) + 16\sec^4\left(x + \frac{\pi}{4}\right)\tan^2\left(x + \frac{\pi}{4}\right)$

when $x = \frac{\pi}{12}, \varphi''(x) = \frac{4}{\left(\frac{1}{2}\right)^6} + \frac{16}{\left(\frac{1}{2}\right)^4}\left(\sqrt{3}\right)^2 = 1024$

42 B

$y = \pi - x$
$dy = -dx$
$\cos y = \cos(\pi - x) = -\cos x$
$x = 0, y = \pi$
$x = \frac{\pi}{2}, y = \frac{\pi}{2}$

$\int_{\pi}^{\pi/2} \cos y \, dy = \int_{\pi}^{\pi/2} \cos x \, dx = \left[\sin x\right]_{\pi}^{\pi/2}$

43 C

$\int_1^{\infty} x^{-3/2} dx$

$= \left[-\frac{2}{\sqrt{x}}\right]_1^{\infty}$

$= 2$

44 C

$\int_0^c 4(2x+1)^3 dx = \frac{15}{2}$

$\frac{1}{2}\left[(2x+1)^4\right]_0^c = \frac{15}{2}$

$(2c+1)^4 - 1 = 15$

$2c + 1 = \sqrt[4]{16} = 2$

$c = \frac{1}{2}$

45 D

$\frac{dy}{dx} = \frac{x^2}{y^2}$

$\int y^2 dy = \int x^2 dx$

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$$\frac{1}{3}y^3 = \frac{1}{3}x^3 + c$$
$$x = 2, y = 0 \Rightarrow c = -8/3$$
$$y^3 = x^3 - 8$$

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