Chapter 9 Trigonometry

Try these 9.1

(a) $\sin 4x = 0.28$

$4x = \sin^{-1}(0.28)$

$4x = 180°n + (-1)^n (16.3°),\ n \in \mathbb{Z}$

$x = \frac{1}{4}[180n + (-1)^n (16.3°)],\ n \in \mathbb{Z}$

(b) $\cos(x + 30°) = 0.6$

$x + 30 = \cos^{-1}(0.6)$

$x + 30 = 360n \pm 53.13°$

$\left.\begin{array}{l} x = 360n + 23.13° \\ x = 360n - 83.13° \end{array}\right\} n \in \mathbb{Z}$

(c) $\tan(2x + 45°) = 0.7$

$2x + 45° = \tan^{-1}(0.7)$

$2x + 45 = 180°n + 35°$

$2x = 180n - 10$

$x = 90°n - 5°,\quad n \in \mathbb{Z}$

Hence $x = 90n - 5°,\quad n \in \mathbb{Z}$

Exercise 9A

1 $\sin 2\theta = \frac{-1}{2}$

$2\theta = n\pi + (-1)^n \left(\frac{-\pi}{6}\right),\ n \in \mathbb{Z}$

$n = 2p \Rightarrow 2\theta = 2p\pi - \frac{\pi}{6}$

$\theta = p\pi - \frac{\pi}{12}$

$n = 2p + 1 \Rightarrow 2\theta = (2p + 1)\pi + \frac{\pi}{6}$

$\theta = \left(\frac{2p+1}{2}\right)\pi + \frac{\pi}{12} = p\pi + \frac{13\pi}{12}$

Hence $\left.\begin{array}{l} \theta = p\pi - \frac{\pi}{12} \\ \quad\ \ p\pi + \frac{13\pi}{12} \end{array}\right\} p \in \mathbb{Z}$

2 $\cos 3\theta = 0$

$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{2},\ n \in Z$

$\theta = \frac{2}{3}n\pi \pm \frac{\pi}{6},\ n \in Z$

3 $\tan\left(2\theta+\frac{\pi}{3}\right)=1$

$$2\theta+\frac{\pi}{3}=n\pi+\frac{\pi}{4}$$

$$2\theta=n\pi+\frac{\pi}{4}-\frac{\pi}{3}$$

$$\theta=\frac{1}{2}\left[n\pi-\frac{\pi}{12}\right]$$

$$\theta=\frac{1}{2}n\pi-\frac{\pi}{24},\ n\in\mathbb{Z}$$

4 $\cos\left(2\theta-\frac{\pi}{4}\right)=\frac{1}{2}$

$$2\theta-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{3}$$

$$2\theta=2n\pi\pm\frac{\pi}{3}+\frac{\pi}{4}$$

$$2\theta=2n\pi+\frac{7\pi}{12},\ \ 2n\pi-\frac{\pi}{12}$$

$$\left.\begin{aligned}\theta&=n\pi+\frac{7\pi}{24}\\ \theta&=n\pi-\frac{\pi}{24}\end{aligned}\right\}n\in\mathbb{Z}$$

5 $\sin\left(3\theta-\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$

$$3\theta-\frac{\pi}{3}=n\pi+(-1)^n\frac{\pi}{4},\ \ n\in\mathbb{Z}$$

$$n=2p]\Rightarrow 3\theta-\frac{\pi}{3}=2p\pi+\frac{\pi}{4}$$

$$3\theta=2p\pi+\frac{\pi}{4}+\frac{\pi}{3}$$

$$\theta=\frac{1}{3}\left[2p\pi+\frac{7\pi}{12}\right],p\in\mathbb{Z}$$

$$n=2p+1]\Rightarrow 3\theta-\frac{\pi}{3}=(2p+1)\,\pi-\frac{\pi}{4}$$

$$3\theta=(2p+1)\,\pi-\frac{\pi}{4}+\frac{\pi}{3}$$

$$3\theta=2p\pi+\pi-\frac{\pi}{4}+\frac{\pi}{3}$$

$$\theta=\frac{1}{3}\left[2p\pi+\frac{13}{12}\pi\right],p\in\mathbb{Z}$$

$$\left.\begin{aligned}\therefore\ \theta&=\frac{1}{3}\left[2p\pi+\frac{7\pi}{12}\right]\\ &\ \ \frac{1}{3}\left[2p\pi+\frac{13\pi}{12}\right]\end{aligned}\right\}p\in\mathbb{Z}$$

6 $\tan\left(\frac{1}{2}\theta - \frac{\pi}{2}\right) = -1$

$\frac{1}{2}\theta - \frac{\pi}{2} = n\pi - \frac{\pi}{4}$

$\frac{1}{2}\theta = n\pi - \frac{\pi}{4} + \frac{\pi}{2}$

$\theta = 2\left[n\pi + \frac{\pi}{4}\right], n \in \mathbb{Z}$

7 sin 3x = 3 cos 3x

$\Rightarrow \frac{\sin 3x}{\cos 3x} = 3$

tan 3x = 3

3x = nπ + 1.25

$x = \frac{1}{3}n\pi + 0.416,\ n \in \mathbb{Z}$

Try these 9.2

(a) RTP: $\frac{\sec\theta - \cos\theta}{\sec\theta + \cos\theta} = \frac{\sin^2\theta}{1+\cos^2\theta}$

Proof:

$$\frac{\sec\theta - \cos\theta}{\sec\theta + \cos\theta} = \frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{1}{\cos\theta} + \cos\theta}$$

$$= \frac{\frac{1-\cos^2\theta}{\cancel{\cos\theta}}}{\frac{1+\cos^2\theta}{\cancel{\cos\theta}}}$$

$$= \frac{1-\cos^2\theta}{1+\cos^2\theta}$$

$$= \frac{\sin^2\theta}{1+\cos^2\theta}$$

(b) RTP: $\frac{1-\cos^2\theta}{\sin\theta} = \sin\theta$

Proof:

$\frac{1-\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta}$

= sin θ

(c) RTP: $\frac{\sec\theta + \tan\theta}{\cot\theta + \cos\theta} = \frac{\sin\theta}{\cos^2\theta}$

Proof:

$\frac{\sec\theta + \tan\theta}{\cot\theta + \cos\theta}$

$$= \frac{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$$

$$= \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\cos\theta + \cos\theta\sin\theta}{\sin\theta}}$$

$$= \frac{(1+\sin\theta)\sin\theta}{(\cos\theta + \cos\theta\sin\theta)\cos\theta}$$

$$= \frac{\cancel{(1+\sin\theta)}\sin\theta}{\cos^2\theta\,\cancel{(1+\sin\theta)}}$$

$$= \frac{\sin\theta}{\cos^2\theta}$$

Try these 9.3

(a) (i) $3\sin^2\theta = 1 + \cos\theta$

$3(1 - \cos^2\theta) = 1 + \cos\theta$

$3\cos^2\theta + \cos\theta - 2 = 0$

$(3\cos\theta - 2)(\cos\theta + 1) = 0$

$\cos\theta = \frac{2}{3}, \cos\theta = -1$

$\theta = 48.2°, 311.8°, 180°$

(ii) $4\operatorname{cosec}^2\theta - 4\cot\theta - 7 = 0$

$4(1 + \cot^2\theta) - 4\cot\theta - 7 = 0$

$4\cot^2\theta - 4\cot\theta - 3 = 0$

$(2\cot\theta + 1)(2\cot\theta - 3) = 0$

$\cot\theta = -\frac{1}{2}, \cot\theta = \frac{3}{2}$

$\tan\theta = -2, \tan\theta = \frac{2}{3}$

$\theta = 116.6°, 296.6°, 33.7°, 213.7°$

(b) $20\sec^2\theta - 3\tan\theta - 22 = 0$

$20(1 + \tan^2\theta) - 3\tan\theta - 22 = 0$

$20\tan^2\theta - 3\tan\theta - 2 = 0$

$(4\tan\theta + 1)(5\tan\theta - 2) = 0$

$\tan\theta = -\frac{1}{4}, \tan\theta = \frac{2}{5}$

$\tan\theta = -\frac{1}{4} \Rightarrow \theta = 180n - 14°, n \in \mathbb{Z}$

$\tan\theta = \frac{2}{5} \Rightarrow \theta = 180n + 21.8, n \in \mathbb{Z}$

Exercise 9B

1 $(\sin\theta + \cos\theta)^2 - 1$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1$

$= 1 + 2\sin\theta\cos\theta - 1$, since $\sin^2\theta + \cos^2\theta = 1$

$= 2\sin\theta\cos\theta$

2 $\sin x(\sin x - \cot x \operatorname{cosec} x)$

$$= \sin^2 x - \sin x\frac{\cos x}{\sin x}\times\frac{1}{\sin x}$$

$$= \sin^2 x - \frac{\cos x}{\sin x}$$

$= \sin^2 x - \cot x$

3 $\sin^4\theta - \cos^4\theta$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

4 $\sin^2\theta(\cot^2\theta + \operatorname{cosec}^2\theta)$

$$= \sin^2\theta\left[\frac{\cos^2\theta}{\sin^2\theta} + \frac{1}{\sin^2\theta}\right]$$

$$= \sin^2\theta\left[\frac{\cos^2\theta + 1}{\sin^2\theta}\right]$$

$= 1 + \cos^2\theta$

5 $$\frac{\sin^2\theta - 1}{\cos^2\theta} = \frac{-(1-\sin^2\theta)}{\cos^2\theta}$$

$$= \frac{-(\cos^2\theta)}{\cos^2\theta} = -1$$

6 $$\frac{\sec^2\theta - \tan^2\theta}{\sin\theta} = \frac{\sec^2\theta - (\sec^2\theta - 1)}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$= \operatorname{cosec}\theta$

7 RTP: $1 - \dfrac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

Proof:

$$1 - \frac{\sin^2\theta}{1-\cos\theta}$$

$$= 1 - \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= 1 - \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$

$= 1 - (1 + \cos\theta)$

$= -\cos\theta$

8 RTP: $\dfrac{\cos^2\theta}{1-\sin\theta} - 1 = \sin\theta$

Proof:

$$\frac{\cos^2\theta}{1-\sin\theta}-1=\frac{1-\sin^2\theta}{1-\sin\theta}-1$$

$$=\frac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta}-1$$

$= 1 + \sin\theta - 1$

$= \sin\theta$

9 RTP: $\dfrac{\operatorname{cosec}\theta}{\cos\theta}-\dfrac{\cos\theta}{\sin\theta}=\tan\theta$

Proof:

$$\frac{\operatorname{cosec}\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin\theta\cos\theta}-\frac{\cos\theta}{\sin\theta}$$

$$=\frac{1-\cos^2\theta}{\sin\theta\cos\theta}$$

$$=\frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}\cos\theta}=\frac{\sin\theta}{\cos\theta}$$

$= \tan\theta$

10 RTP: $\dfrac{1-\cos^2\theta}{1-\sec^2\theta}=-\cos^2\theta$

Proof:

$$\frac{1-\cos^2\theta}{1-\sec^2\theta}=\frac{\sin^2\theta}{-\tan^2\theta}$$

$$=\cancel{\sin^2\theta}\times-\frac{\cos^2\theta}{\cancel{\sin^2\theta}}$$

$= -\cos^2\theta$

11 RTP: $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Proof:

$\sec^4 x - \sec^2 x$

$= \sec^2 x(\sec^2 x - 1)$

$= (1 + \tan^2 x)(\tan^2 x)$, since $\sec^2 x - 1 = \tan^2 x$

$= \tan^2 x + \tan^4 x$

12 RTP: $\dfrac{\cos x}{1-\tan x}+\dfrac{\sin x}{1-\cot x}=\sin x+\cos x$

Proof:

$$\frac{\cos x}{1-\tan x}+\frac{\sin x}{1-\cot x}$$

$$=\frac{\cos x}{1-\dfrac{\sin x}{\cos x}}+\frac{\sin x}{1-\dfrac{\cos x}{\sin x}}$$

$$=\frac{\cos x}{\dfrac{\cos x-\sin x}{\cos x}}+\frac{\sin x}{\dfrac{\sin x-\cos x}{\sin x}}$$

$= \dfrac{\cos^2 x}{\cos x - \sin x} + \dfrac{\sin^2 x}{\sin x - \cos x}$

$= \dfrac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$

$= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$

$= \cos x + \sin x$

13 RTP: $\dfrac{1 - \cos x}{1 + \cos x} = (\operatorname{cosec} x - \cot x)^2$

Proof:

Now $(\operatorname{cosec} x - \cot x)^2$

$= \left(\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}\right)^2$

$= \left(\dfrac{1 - \cos x}{\sin x}\right)^2$

$= \dfrac{(1 - \cos x)^2}{\sin^2 x}$

$= \dfrac{(1 - \cos x)^2}{1 - \cos^2 x}$

$= \dfrac{(1 - \cos x)(1 - \cos x)}{(1 - \cos x)(1 + \cos x)}$

$= \dfrac{1 - \cos x}{1 + \cos x}$

14 RTP: $\dfrac{1}{\sin x + 1} + \dfrac{1}{1 - \sin x} = 2\sec^2 x$

Proof:

$\dfrac{1}{\sin x + 1} + \dfrac{1}{1 - \sin x} = \dfrac{1 - \sin x + 1 + \sin x}{(1 + \sin x)(1 - \sin x)}$

$= \dfrac{2}{1 - \sin^2 x}$

$= \dfrac{2}{\cos^2 x}$

$= 2\sec^2 x$

15 RTP: $\sin^4\theta - \sin^2\theta = \cos^4\theta - \cos^2\theta$

Proof:

$\sin^4\theta - \sin^2\theta$

$= \sin^2\theta(\sin^2\theta - 1)$

$= (1 - \cos^2\theta)(-\cos^2\theta) = -\cos^2\theta + \cos^4\theta$

16 RTP: $\dfrac{\cot^2 x - 1}{\tan^2 x - 1} = -\cot^2 x$

Proof:

$$\frac{\cot^2 x - 1}{\tan^2 x - 1} = \frac{\dfrac{\cos^2 x}{\sin^2 x} - 1}{\dfrac{\sin^2 x}{\cos^2 x} - 1}$$

$$= \frac{\cancel{\cos^2 x - \sin^2 x}}{\sin^2 x} \times \frac{\cos^2 x}{\underset{-1}{\cancel{\sin^2 x - \cos^2 x}}}$$

$$= -\frac{\cos^2 x}{\sin^2 x}$$

$= -\cot^2 x$

17 $4 \sec x - \tan x = 6 \cos x$

Converting to sin x and cos x:

$$\frac{4}{\cos x} - \frac{\sin x}{\cos x} = 6 \cos x$$

$\Rightarrow 4 - \sin x = 6 \cos^2 x$

$\Rightarrow 4 - \sin x = 6 [1 - \sin^2 x]$

$4 - \sin x = 6 - 6 \sin^2 x$

$6 \sin^2 x - \sin x - 2 = 0$

Let $y = \sin x$

$6y^2 - y - 2 = 0$

$(3y - 2)(2y + 1) = 0$

$$y = \frac{2}{3}, -\frac{1}{2}$$

$$\sin x = \frac{2}{3}, \sin x = -\frac{1}{2}$$

$x = 41.8°, 138.2°, x = 210°, 330°$

$\therefore\ x = 41.8°, 138.2°, 210°, 330°$

18 $3 \tan^2 x - \sec x - 1 = 0$

Replacing $\tan^2 x = \sec^2 x - 1$

$\Rightarrow 3 [\sec^2 x - 1] - \sec x - 1 = 0$

$3 \sec^2 x - \sec x - 4 = 0$

$y = \sec x$

$3y^2 - y - 4 = 0$

$(3y - 4)(y + 1) = 0$

$$y = \frac{4}{3}, -1$$

$$\text{Now } \sec x = \frac{4}{3}, \sec x = -1$$

$$\Rightarrow \cos x = \frac{3}{4} \qquad \cos x = -1$$

$x = 41.4°, 318.6°, x = 180°$

$\therefore\ x = 41.4°, 180°, 318.6°$

19 $2 \cot^2 x + \operatorname{cosec} x = 1$

Replacing $\cot^2 x = \operatorname{cosec}^2 x - 1$

$\Rightarrow 2 [\operatorname{cosec}^2 x - 1] + \operatorname{cosec} x = 1$

$\Rightarrow 2 \operatorname{cosec}^2 x + \operatorname{cosec} x - 3 = 0$

$y = \operatorname{cosec} x$

$2y^2 + y - 3 = 0$

$\Rightarrow (2y + 3)(y - 1) = 0$

$y = -\frac{3}{2}, y = 1$

$\text{cosec } x = \frac{-3}{2}, \text{cosec } x = 1$

$\sin x = \frac{-2}{3}, \sin x = 1$

$x = 221.8°, 318.2°, x = 90°$

$x = 90°, 221.8°, 318.2°$

20 $3\cos^2 x = 4\sin x - 1$

$\Rightarrow 3(1 - \sin^2 x) = 4\sin x - 1$

$\therefore\ 3\sin^2 x + 4\sin x - 4 = 0$

$y = \sin x$

$3y^2 + 4y - 4 = 0$

$(3y - 2)(y + 2) = 0$

$y = \frac{2}{3}, -2$

$\therefore \sin x = \frac{2}{3}, \sin x = -2$ (invalid)

$x = 41.8°, 138.2°$

21 $2\cot x = 3\sin x$

$\Rightarrow \frac{2\cos x}{\sin x} = 3\sin x$

$\Rightarrow 2\cos x = 3\sin^2 x$

$2\cos x = 3(1 - \cos^2 x)$

$3\cos^2 x + 2\cos x - 3 = 0$

$\cos x = \frac{-2 \pm \sqrt{40}}{6}$

$\cos x = -1.387, 0.72076$

$\text{cox } x = -1.387$ (invalid)

$\cos x = 0.72076$

$\Rightarrow x = 43.9°, 316.1°$

22 $\sin^2 x = 3\cos^2 x + 4\sin x$

$\cos^2 x = 1 - \sin^2 x$

$\therefore\ \sin^2 x = 3(1 - \sin^2 x) + 4\sin x$

$\Rightarrow \sin^2 x = 3 - 3\sin^2 x + 4\sin x$

$4\sin^2 x - 4\sin x - 3 = 0$

$(2\sin x + 1)(2\sin x - 3) = 0$

$\sin x = -\frac{1}{2}, \sin x = \frac{3}{2}$ (invalid)

$x = 210°, 330°$

23 $2\cos x = \tan x$

$2\cos x = \frac{\sin x}{\cos x}$

$\Rightarrow 2\cos^2 x = \sin x$

$\Rightarrow 2[1 - \sin^2 x] = \sin x$

$2\sin^2 x + \sin x - 2 = 0$

$\sin x = \frac{-1 \pm \sqrt{17}}{4}$

$\therefore \sin x = \dfrac{-1 \pm \sqrt{17}}{4}$

$x = 51.3°, 128.7°$

24 $2 \cot x = 1 + \tan x$

$\dfrac{2 \cos x}{\sin x} = 1 + \dfrac{\sin x}{\cos x}$

$\Rightarrow \dfrac{2}{\tan x} = 1 + \tan x$

$2 = \tan x + \tan^2 x$

$\therefore \tan^2 x + \tan x - 2 = 0$

$(\tan x - 1)(\tan x + 2) = 0$

$\tan x = 1, \tan x = -2$

$x = 45°, 225°, x = 116.6°, 296.6°$

25 $2 + 3 \sin z = 2 \cos^2 z$

$2 + 3 \sin z = 2(1 - \sin^2 z)$

$2 \sin^2 z + 3 \sin z = 0$

$\sin z (2 \sin z + 3) = 0$

$\sin z = 0, \sin z = \dfrac{-3}{2}$ (invalid)

$z = n\pi + (-1)^n (0)$

$z = n\pi,\ n \in \square$

26 $2 \cot^2 x + \operatorname{cosec} x = 4$

$\Rightarrow 2(\operatorname{cosec}^2 x - 1) + \operatorname{cosec} x - 4 = 0$

$\Rightarrow 2 \operatorname{cosec}^2 x + \operatorname{cosec} x - 6 = 0$

$\Rightarrow (2 \operatorname{cosec} x - 3)(\operatorname{cosec} x + 2) = 0$

$\operatorname{cosec} x = \dfrac{3}{2}, \operatorname{cosec} x = -2$

$\therefore \sin x = \dfrac{2}{3}, \sin x = -\dfrac{1}{2}$

$x = n\pi + (-1)^n (0.730)\ \ n \in \square$

$x = n\pi + (-1)^n \left(\dfrac{-\pi}{6}\right), n \in \square$

27 $2 \sec x + 3 \cos x = 7$

$\dfrac{2}{\cos x} + 3 \cos x = 7$

$2 + 3 \cos^2 x = 7 \cos x$

$3 \cos^2 x - 7 \cos x + 2 = 0$

$(3 \cos x - 1)(\cos x - 2) = 0$

$\cos x = \dfrac{1}{3}, \cos x = 2$ (invalid)

$x = 2n\pi \pm 1.23,\ n \in \square$

(28) $5 \cos x = 6 \sin^2 x$

$5 \cos x = 6(1 - \cos^2 x)$

$6 \cos^2 x + 5 \cos x - 6 = 0$

$(3 \cos x - 2)(2 \cos x + 3) = 0$

$\cos x = \dfrac{2}{3},\ \cos x = \dfrac{-3}{2}$ (invalid)

$x = 2n\pi \pm (0.841)\ \ n \in \square$

Try these 9.4

(a) $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2}}{4}(1 - \sqrt{3})$

(b) $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

(c) $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

Exercise 9C

1 sin 75 = sin (30 + 45)
= sin 30 cos 45 + cos 30 sin 45

$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

2 $\frac{\sin(A - B)}{\sin(A + B)} = \frac{5}{13}$

$$\frac{\sin A\cos B - \cos A\sin B}{\sin A\cos B + \cos A\sin B} = \frac{5}{13}$$

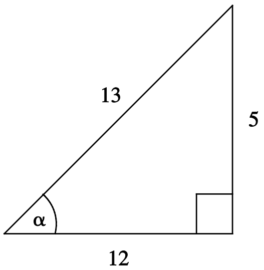
13 sin A cos B – 13 cos A sin B = 5 sin A cos B + 5 cos A sin B

18 cos A sin B = 8 sin A cos B

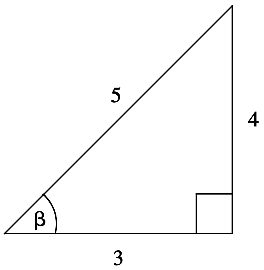
$$18\frac{\sin B}{\cos B} = 8\frac{\sin A}{\cos A}$$

9 tan B = 4 tan A

3 $\tan\alpha = \frac{5}{12}$



$\cos\beta = -\frac{3}{5}$



(a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$= \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{-12}{13}\right)\left(\frac{4}{5}\right)$$

$$= \frac{15}{65} - \frac{48}{65} = \frac{-33}{65}$$

(b) $$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{5}{12} - \left(\frac{-4}{3}\right)}{1+\left(\frac{5}{12}\right)\left(\frac{-4}{3}\right)} = \frac{63}{16}$$

(c) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$= \left(\frac{-12}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{4}{5}\right) = \frac{16}{65}$$

4 $\sin(\theta+30) + \sqrt{3}\cos(\theta+30)$

$$= \sin\theta\cos 30 + \cos\theta\sin 30 + \sqrt{3}\cos\theta\cos 30 - \sqrt{3}\sin\theta\sin 30$$

$$= \cancel{\frac{\sqrt{3}}{2}\sin\theta} + \frac{1}{2}\cos\theta + \sqrt{3}\left(\frac{\sqrt{3}}{2}\cos\theta\right) - \cancel{\frac{\sqrt{3}}{2}\sin\theta}$$

$$= \frac{1}{2}\cos\theta + \frac{3}{2}\cos\theta$$

= 2 cosθ.

5 sin (θ + 30) = 2 cos (θ + 60)

$\sin\theta\cos 30 + \cos\theta\sin 30 = 2\cos\theta\cos 60 - 2\sin\theta\sin 60$

$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{1}{2}(2)\cos\theta - 2\frac{\sqrt{3}}{2}\sin\theta$

$\frac{\sqrt{3}}{2}\sin\theta + \sqrt{3}\sin\theta = \cos\theta - \frac{1}{2}\cos\theta$

$\frac{3\sqrt{3}}{2}\sin\theta = \frac{1}{2}\cos\theta$

$3\sqrt{3}\sin\theta = \cos\theta$

6 $\sin(\theta + \alpha) = k\sin(\theta - \alpha)$

$\sin\theta\cos\alpha + \cos\theta\sin\alpha = k\sin\theta\cos\alpha - k\cos\theta\sin\alpha$

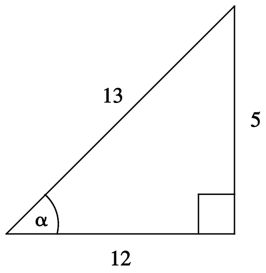
$\cos\theta\sin\alpha + k\cos\theta\sin\alpha = k\sin\theta\cos\alpha - \sin\theta\cos\alpha$

$\cos\theta\sin\alpha(k+1) = \sin\theta\cos\alpha(k-1)$

$\frac{\sin\alpha}{\cos\alpha}\left[\frac{k+1}{k-1}\right] = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \left(\frac{k+1}{k-1}\right)\tan\alpha$

7 $\cos\alpha = \frac{-12}{13}$



(a) $\sin\alpha = \frac{+5}{13}$

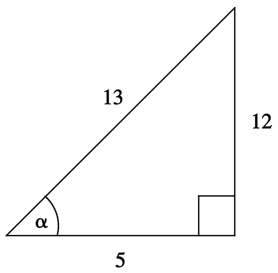
(b) $\tan\alpha = \frac{-5}{12}$

(c) $\cos(\alpha + 30) = \cos\alpha\cos 30 - \sin\alpha\sin 30$

$= \left(\frac{-12}{13}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{5}{13}\right)\left(\frac{1}{2}\right)$

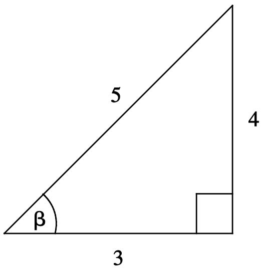
$= \frac{-12\sqrt{3}}{26} - \frac{5}{26} = \frac{-1}{26}(5 + 12\sqrt{3})$

8 $\sin\alpha = \frac{12}{13}$



$\therefore \cos\alpha = \frac{-5}{13}$

$\sin\beta = \frac{4}{5}$



$\cos\beta = \frac{-3}{5}$

both α and β are in the second quadrant

(a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{12}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{4}{5}\right)$

$= \frac{-36}{65}\frac{-20}{65} = \frac{-56}{65}$

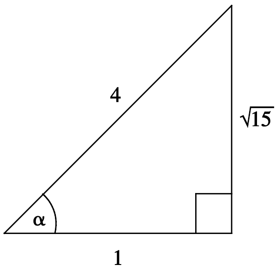
(b) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$

$= \frac{15}{65} - \frac{48}{65} = \frac{-33}{65}$

(c) $\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\frac{-56}{65}}{\frac{-33}{65}} = \frac{56}{33}$

9 $\cos\alpha = \frac{1}{4}$



Since α is in the 4th quadrant sin α is negative

(a) $\sin\alpha = \frac{-\sqrt{15}}{4}$

(b) $\sin\left(\alpha - \frac{\pi}{6}\right) = \sin\alpha\cos\frac{\pi}{6} - \cos\alpha\sin\frac{\pi}{6}$

$= \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)$

$= \frac{-3\sqrt{5}}{8} - \frac{1}{8}$

(c) $\cos\left(\alpha+\frac{\pi}{3}\right)=\cos\alpha\cos\frac{\pi}{3}-\sin\alpha\sin\frac{\pi}{3}$

$=\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)-\left(\frac{-\sqrt{15}}{4}\right)\left(\frac{\sqrt{3}}{2}\right)$

$=\frac{1}{8}+\frac{3\sqrt{5}}{8}$

10 (a) $\tan(2\pi-\theta)$

$=\frac{\tan 2\pi-\tan\theta}{1+\tan 2\pi\tan\theta}=\frac{0-\tan\theta}{1+0}$

$=-\tan\theta$

(b) $\sin\left(\frac{3\pi}{2}+\theta\right)$

$=\sin\left(\frac{3\pi}{2}\right)\cos\theta+\cos\left(\frac{3\pi}{2}\right)\sin\theta$

$=(-1)\cos\theta+(0)\sin\theta$

$=-\cos\theta$

11 $\tan A=y+1$

$\tan B=y-1$

$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$=\frac{y+1-(y-1)}{1+(y+1)(y-1)}$

$=\frac{2}{1+y^2-1}$

$=\frac{2}{y^2}$

$2\cot(A-B)=\frac{2}{\tan(A-B)}$

$=\frac{2}{\frac{2}{y^2}}$

$=y^2$

12 (a) $\frac{1+\tan\theta}{1-\tan\theta}$

$=\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}$

$=\tan\left(\frac{\pi}{4}+\theta\right)$

(b) $\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta$

$=\cos\frac{\pi}{4}\cos\theta+\sin\frac{\pi}{4}\sin\theta$

$= \cos\left(\frac{\pi}{4} - \theta\right)$

13 $\cot(\theta - \alpha) = 4$

$\Rightarrow \tan(\theta - \alpha) = \frac{1}{4}$

$\Rightarrow \frac{\tan\theta - \tan\alpha}{1 - \tan\theta\tan\alpha} = \frac{1}{4} \quad \cot\alpha = \frac{1}{2} \Rightarrow \tan\alpha = 2$

$\Rightarrow \frac{\tan\theta - 2}{1 - 2\tan\theta} = \frac{1}{4}$

$\Rightarrow 4\tan\theta - 8 = 1 - 2\tan\theta$

$6\tan\theta = 9$

$\tan\theta = \frac{9}{6} = \frac{3}{2}$

$\Rightarrow \cot\theta = \frac{2}{3}$

14 $\frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{\sin(\alpha + \beta) + \sin(\alpha - \beta)}$

$= \frac{\cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta - \cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}}$

$= \frac{\cancel{2\sin\alpha}\sin\beta}{\cancel{2\sin\alpha}\cos\beta}$

$= \frac{\sin\beta}{\cos\beta}$

$= \tan\beta$

15 $\tan(\alpha + \beta) = b$

$\tan\beta = \frac{1}{2}$

$\tan(\alpha + \beta) = b \qquad \Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = b$

$\Rightarrow \frac{\tan\alpha + \frac{1}{2}}{1 - \frac{1}{2}\tan\alpha} = b$

$\frac{2\tan\alpha + 1}{2 - \tan\alpha} = b$

$\Rightarrow 2\tan\alpha + 1 = 2b - b\tan\alpha$

$\Rightarrow 2\tan\alpha + b\tan\alpha = 2b - 1$

$\tan\alpha(2 + b) = 2b - 1$

$\tan\alpha = \frac{2b - 1}{b + 2}$

16 $P = VI\cos\phi\sin^2(\omega t) - VI\sin\phi\sin\omega t\cos\omega t$

$= VI\sin\omega t[\cos\phi\sin\omega t - \sin\phi\cos\omega t]$

$= VI\sin\omega t\sin(\omega t - \phi)$

Try these 9.5

(a) $3\sin\theta - \cos\theta = r\sin(\theta - \alpha)$

$= r\sin\theta\cos\alpha - r\cos\theta\sin\alpha$

$\Rightarrow r\cos\alpha = 3$ [1]

$r\sin\alpha = 1$ [2]

$[2] \div [1] \Rightarrow \tan\alpha = \frac{1}{3}, \quad \alpha = 18.4°$

$[1]^2 + [2]^2 \Rightarrow r^2[\cos^2\alpha + \sin^2\alpha] = 3^2 + 1^2$

$r^2 = 10$

$r = \sqrt{10}$

$\therefore 3\sin\theta - \cos\theta = \sqrt{10}\sin(\theta - 18.4°)$

$\sqrt{10}\sin(\theta - 18.4°) = 2$

$\sin(\theta - 18.4) = \frac{2}{\sqrt{10}}$

$\theta - 18.4 = \sin^{-1}\left(\frac{2}{\sqrt{10}}\right)$

$\theta - 18.4 = 39.2°, 140.8°$

$\theta = 57.6°, 159.2°$

(b) $\sqrt{3}\cos 2\theta - \sin 2\theta = r\cos(2\theta + \alpha)$

$= r\cos 2\theta\cos\alpha - r\sin 2\theta\sin\alpha$

$\Rightarrow r\cos\alpha = \sqrt{3}$ [1]

$r\sin\alpha = 1$ [2]

$[2] \div [1] \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}} \quad \Rightarrow \alpha = 30°$

$[1]^2 + [2]^2] \quad \Rightarrow r^2 = 3 + 1 \Rightarrow r = 2$

$\therefore \sqrt{3}\cos 2\theta - \sin 2\theta = 2\cos(2\theta + 30°)$

$2\cos(2\theta + 30°) = -1$

$\cos(2\theta + 30°) = \frac{-1}{2}$

$2\theta + 30° = 360n \pm 120°$

$2\theta = 360n + 90°$

$2\theta = 360n - 150$

$$\Rightarrow \theta = \left.\begin{array}{l}180n + 45° \\ 180n - 75°\end{array}\right\} n \in \square$$

(c) $2\sin x - \cos x = r\sin(x - \alpha)$

$= r\sin x\cos\alpha - r\cos x\sin\alpha$

$r\cos\alpha = 2$

$r\sin\alpha = 1$

$\tan\alpha = \frac{1}{2} \Rightarrow \alpha = 26.6°$

$r^2 = 2^2 + 1^2 \Rightarrow r = \sqrt{5}$

$2\sin x - \cos x = \sqrt{5}\sin(x - 26.6°)$

$\max f(x) = \sqrt{5}$

when $\sin(x - 26.6) = 1$, $x - 26.6 = 90$

$x = 116.6^{\circ}$

$\min f(x) = -\sqrt{5}$

when $\sin(x - 26.6) = -1$

$x - 26.6 = 270$

$x = 296.6^{\circ}$

Try these 9.6

(a) $\cos 2\theta - \cos\theta = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$, $\cos\theta = 1$

$\theta = 120°, 240°, 0°, 360°$

Hence $\theta = 0°, 120°, 240°, 360°$

(b) $\cos\theta = \sin 2\theta$

$\cos\theta - 2\sin\theta\cos\theta = 0$

$\cos\theta(1 - 2\sin\theta) = 0$

$\cos\theta = 0$, $\sin\theta = \frac{1}{2}$

$\theta = 0°, 180°, \theta = 30°, 150°$

Hence $\theta = 0°, 30°, 150°, 180°$

(c) $\cos 2\theta - 2\cos\theta = 3$

$2\cos^2\theta - 1 - 2\cos\theta - 3 = 0$

$2\cos^2\theta - 2\cos\theta - 4 = 0$

$\cos^2\theta - \cos\theta - 2 = 0$

$(\cos\theta + 1)(\cos\theta - 2) = 0$

$\cos\theta = -1$, $\cos\theta = 2$.

$\theta = 180°$ $\quad$ $\cos\theta = 2$ has no solutions

Hence $\theta = 180°$

Exercise 9D

1 RTP: $\dfrac{\sin 2x + \cos x}{2 - 2\cos^2 x + \sin x} = \cot x$

Proof:

$$\frac{\sin 2x + \cos x}{2 - 2\cos^2 x + \sin x}$$

$$= \frac{2\sin x\cos x + \cos x}{2\sin^2 x + \sin x}$$

$$= \frac{\cos x\,[2\sin x + 1]}{\sin x\,[2\sin x + 1]}$$

$= \dfrac{\cos x}{\sin x}$

$= \cot x$

2 RTP: $\dfrac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$

Proof:

$\dfrac{1 - \cos 2x}{1 + \cos 2x}$

$= \dfrac{\cancel{2} \sin^2 x}{\cancel{2} \cos^2 x}$

$= \tan^2 x$

3 RTP: $\tan x - \cot x = -2 \cot 2x$

Proof:

$\tan x - \cot x = \dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin^2 x - \cos^2 x}{\cos x \sin x} = \dfrac{-[\cos^2 x - \sin^2 x]}{\frac{1}{2} \sin 2x} = \dfrac{-2 \cos 2x}{\sin 2x} = -2 \cot 2x$

4 RTP: $\dfrac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$

Proof:

$\dfrac{\cos 2x}{\cos x - \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$

$= \dfrac{\cancel{(\cos x - \sin x)}\ (\cos x + \sin x)}{\cancel{\cos x - \sin x}}$

$= \cos x + \sin x$

5 RTP: $\dfrac{1 - \cos 2A + \sin A}{\sin 2A + \cos A} = \tan A$

Proof:

$\dfrac{1 - \cos 2A + \sin A}{\sin 2A + \cos A}$

$= \dfrac{2 \sin^2 A + \sin A}{2 \sin A \cos A + \cos A}$

$= \dfrac{\sin A\ \cancel{(2 \sin A + 1)}}{\cos A\ \cancel{(2 \sin A + 1)}}$

$= \dfrac{\sin A}{\cos A}$

$= \tan A$

6 RTP: $\dfrac{1 - \cos 4\theta}{\sin 4\theta} = \tan 2\theta$

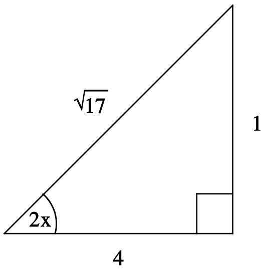
Proof:

$\dfrac{1 - \cos 4\theta}{\sin 4\theta} = \dfrac{\cancel{2} \sin^{\cancel{2}} 2\theta}{\cancel{2}\ \cancel{\sin 2\theta}\ \cos 2\theta}$

$$= \frac{\sin 2\theta}{\cos 2\theta}$$

$= \tan 2\theta$

7 $\tan 2x = \frac{1}{4}$



(a) $\cos 2x = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$

(b) $\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$$\sin^2 x = \frac{1}{2}\left[1 - \frac{4}{\sqrt{17}}\right]$$

$$\sin x = \sqrt{\frac{1}{2} - \frac{2}{\sqrt{17}}}$$

$$= \sqrt{\frac{1}{2} - \frac{2\sqrt{17}}{17}}$$

8 RTP: $\frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \tan y$

Proof:

$$\frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \frac{\cancel{\sin x \cos y} + \cos x \sin y - \cancel{\sin x \cos y} + \cos x \sin y}{\cos x \cos y - \cancel{\sin x \sin y} + \cos x \cos y + \cancel{\sin x \sin y}}$$

$$= \frac{\cancel{2\cos x} \sin y}{\cancel{2\cos x} \cos y} = \frac{\sin y}{\cos y} = \tan y$$

9 $\frac{\sin(\theta - \alpha)}{\sin(\theta + \alpha)} = \frac{4}{5}$

$5\sin(\theta - \alpha) = 4\sin(\theta + \alpha)$

$\Rightarrow 5\sin\theta\cos\alpha - 5\cos\theta\sin\alpha = 4\sin\theta\cos\alpha + 4\cos\theta\sin\alpha$

$5\sin\theta\cos\alpha - 4\sin\theta\cos\alpha = 5\cos\theta\sin\alpha + 4\cos\theta\sin\alpha$

$\sin\theta\cos\alpha = 9\cos\theta\sin\alpha$

$$\frac{\sin\theta}{\cos\theta} = 9\frac{\sin\alpha}{\cos\alpha}$$

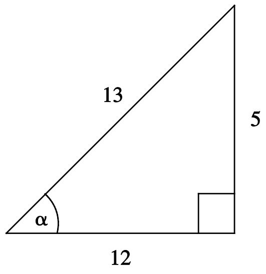
$\tan\theta = 9\tan\alpha$

$$\tan\alpha = \frac{1}{3},\ \tan\theta = (9)\left(\frac{1}{3}\right) = 3$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$= \frac{2(3)}{1-(3)^2} = \frac{6}{-8} = \frac{-3}{4}$$

10 $\tan\alpha = \frac{5}{12}$



$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

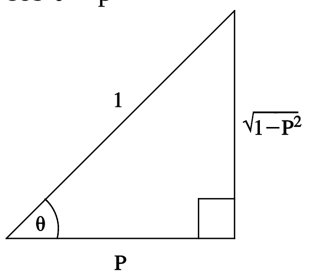
$\cos 2\alpha = 2\cos^2\alpha - 1$

$= 2\left(\frac{12}{13}\right)^2 - 1 = \frac{119}{169}$

$\cos 4\alpha = 2\cos^2(2\alpha) - 1$

$= 2\left(\frac{119}{169}\right)^2 - 1 = \frac{-239}{28\,561}$

11 $\cos\theta = p$



(a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2p\sqrt{1-p^2}$

(b) $\tan^2\theta = \left(\frac{\sqrt{1-p^2}}{p}\right)^2 = \frac{1-p^2}{p^2}$

(c) $\sin 4\theta = 2\sin 2\theta\cos 2\theta$

$= 2\sin 2\theta\,[2\cos^2\theta - 1]$

$= 4p\sqrt{1-p^2}\,[2p^2 - 1]$

12 $\tan 2\alpha = 1$

$\frac{2\tan\alpha}{1-\tan^2\alpha} = 1$

$2\tan\alpha = 1 - \tan^2\alpha$

$\tan^2\alpha + 2\tan\alpha - 1 = 0$

$\tan\alpha = \frac{-2 \pm \sqrt{8}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2}$

$\tan\alpha = -1 + \sqrt{2}, -1 - \sqrt{2}$

Since $0° < \alpha < 90° \Rightarrow \tan\alpha = \sqrt{2} - 1$

13 $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$\alpha = 67\frac{1}{2}$

$\Rightarrow \tan 2\left(67\frac{1}{2}\right) = \dfrac{2\tan 67\frac{1}{2}}{1 - \tan^2 67\frac{1}{2}}$

$\tan 135 = \dfrac{2\tan 67\frac{1}{2}}{1 - \tan^2 67\frac{1}{2}}$

$-1 = \dfrac{2\tan 67\frac{1}{2}}{1 - \tan^2 67\frac{1}{2}}$

$\Rightarrow -1 + \tan^2 67\frac{1}{2} = 2\tan 67\frac{1}{2}$

$\Rightarrow \tan^2 67\frac{1}{2} - 2\tan 67\frac{1}{2} - 1 = 0$

$\tan 67\frac{1}{2} = \dfrac{2 \pm \sqrt{8}}{2}$

$= \dfrac{2 \pm 2\sqrt{2}}{2}$

$= 1 \pm \sqrt{2}$

$\therefore \tan 67\dfrac{1°}{2} = 1 + \sqrt{2}$, since the angle is acute

14 (a) $\tan\theta = \dfrac{3}{4}$

$\tan(\theta + \beta) = -2$

$\dfrac{\tan\theta + \tan\beta}{1 - \tan\theta\tan\beta} = -2$

$\dfrac{\frac{3}{4} + \tan\beta}{1 - \frac{3}{4}\tan\beta} = -2$

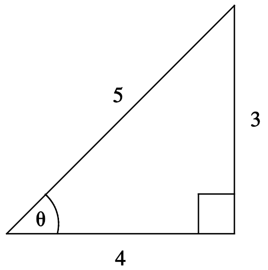
$\dfrac{3}{4} + \tan\beta = -2 + \dfrac{3}{2}\tan\beta$

$\dfrac{3}{4} + 2 = \dfrac{3}{2}\tan\beta - \tan\beta$

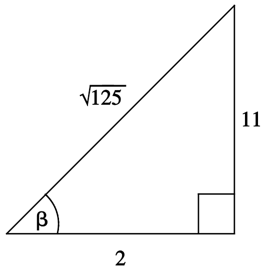
$\dfrac{11}{4} = \dfrac{1}{2}\tan\beta$

$\tan\beta = \dfrac{11}{4} \times 2 = \dfrac{11}{2}$

(b)



$\sin\theta = \frac{3}{5}$



$\sin\beta = \frac{11}{\sqrt{125}}$

$= \frac{11}{5\sqrt{5}}$

$= \frac{11\sqrt{5}}{25}$

15 $\frac{\cos(A - B)}{\cos(A + B)} = \frac{5}{2}$

2 cos(A – B) = 5 cos(A + B)

2 cos A cos B + 2 sin A sin B = 5 cos A cos B – 5 sin A sin B

2 sin A sin B + 5 sin A sin B = 5 cos A cos B – 2 cos A cos B

⇒ 7 sin A sin B = 3 cos A cos B

$\frac{7\sin A}{\cos A} = \frac{3\cos B}{\sin B}$

7 tan A = 3 cot B

tan B = 3

$\cot B = \frac{1}{3}$

$\tan A = \frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$

(a) tan (A + B)

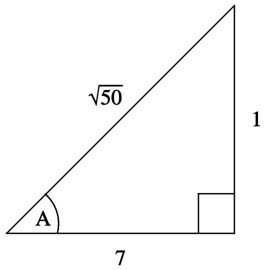
$= \frac{\tan A + \tan B}{1 - \tan A \ \tan B}$

$= \frac{\frac{1}{7} + 3}{1 - \frac{3}{7}}$

$= \frac{\frac{22}{7}}{\frac{4}{7}}$

$= \frac{22}{4} = \frac{11}{2}$

(b)



$\sin A = \frac{1}{\sqrt{50}} = \frac{\sqrt{50}}{50}$

(c) $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{\sqrt{50}}{50}\right)^2$

$= 1 - \frac{2}{50}$

$= \frac{24}{25}$

16 Since α, β and θ are the angles of a triangle:

$\alpha + \beta + \theta = 180°$

$\theta = 180 - (\alpha + \beta)$

$\tan \theta = \tan (180 - (\alpha + \beta))$

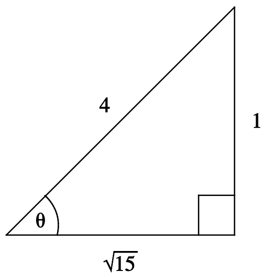
$= \frac{\tan 180 - \tan(\alpha + \beta)}{1 - \tan 180 \tan(\alpha + \beta)}$

$= -\tan(\alpha + \beta)$

$= -\left[\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\right]$

$= \frac{\tan \alpha + \tan \beta}{\tan \alpha \tan \beta - 1}$

17



(a) $\cos 2\theta = 1 - 2 \sin^2 \theta$

$= 1 - 2\left(\frac{1}{4}\right)^2$

$= 1 - \frac{2}{16}$

$= \frac{7}{8}$

(b) $\cos 4\theta = 2\cos^2(2\theta) - 1$

$= 2\left(\frac{7}{8}\right)^2 - 1$

$= \frac{49}{32} - 1$

$= \frac{17}{32}$

18 $3\cos x + 2\sin x = r\cos(x - \alpha)$

$3\cos x + 2\sin x = r[\cos x\cos\alpha + \sin x\sin\alpha]$

Comparing coefficients of cos x and sin x

$\Rightarrow r\cos\alpha = 3$ [1]

$r\sin\alpha = 2$ [2]

$[2] \div [1] \Rightarrow \tan\alpha = \frac{2}{3}$

$\alpha = 33.7°$

$[1]^2 + [2]^2 \Rightarrow r^2 = 3^2 + 2^2$

$r = \sqrt{13}$

$\therefore 3\cos x + 2\sin x = \sqrt{13}\cos(x - 33.7°)$

(a) Max values = $\sqrt{13}$

When $\cos(x - 33.7°) = 1$

$\Rightarrow x - 33.7° = 0$

$x = 33.7°$

(b) $3\cos x + 2\sin x = 2$

$\Rightarrow \sqrt{13}\cos(x - 33.7°) = 2$

$\cos(x - 33.7°) = \frac{2}{\sqrt{13}}$

$x - 33.7° = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$

$x - 33.7° = 360°n \pm 56.3°$

$\left.\begin{array}{l} x = 360°n + 90° \\ x = 360°n - 22.6° \end{array}\right\} n \in \mathbb{Z}$

19 (a) $2\sin x + 4\cos x = r\sin(x + \alpha)$

$2\sin x + 4\cos x = r\sin x\cos\alpha + r\cos x\sin\alpha$

Equating coefficients of sin x and cos x

$\Rightarrow r\cos\alpha = 2$ [1]

$r\sin\alpha = 4$ [2]

$[2] \div [1] \Rightarrow \tan\alpha = \frac{4}{2} = 2 \quad \Rightarrow \alpha = 63.4°$

$[1]^2 + [2]^2 \Rightarrow r^2 = 2^2 + 4^2$

$r = \sqrt{20}$

$\therefore 2\sin x + 4\cos x = \sqrt{20}\sin(x + 63.4°)$

(b) $\text{Max}\left(\frac{2}{2\sin x + 4\cos x}\right)$

$= \text{Max}\left(\frac{2}{\sqrt{20}\sin(x + 63.4°)}\right)$

$= \frac{2\sqrt{20}}{20}$

$= \frac{4\sqrt{5})}{20} = \frac{\sqrt{5}}{5}$

20 4 cos x – 3 sin x = r cos (x + α)

4 cos x – 3 sin x = r cos x cos α – r sin x sin α

(a) Equating coefficients of cos x and sin x

$\Rightarrow$ r cos α = 4 [1]

r sin α = 3 [2]

$[2] \div [1] \Rightarrow \tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.7°$

$[1]^2 + [2]^2 \Rightarrow r^2 = 3^2 + 4^2$

$r = \sqrt{25} = 5$

∴ 4 cos x – 3 sin x = 5 cos (x + 36.7°)

(b) 5 cos (x + 36.9°) = 2

$\cos(x + 36.9°) = \frac{2}{5}$

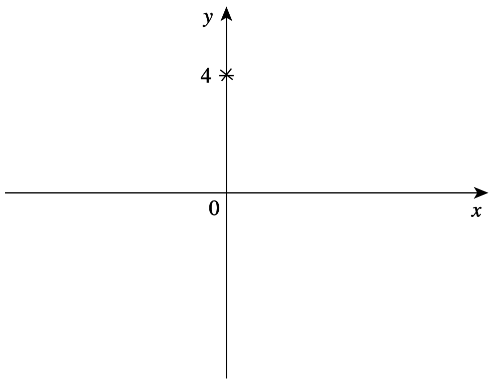
x + 36.9° = 66.4°, 293.6°

x = 29.5°, 256.7°

(c) max (4 – 5 cos (x + 36.9°))

= 4 + 5

= 9



cos(x + 36.9) = 1

x + 36.9 = 0°, 360

x = –36.9, 323.1

Try these 9.7

(a) sin (C + D) = sin C cos D + cos C sin D [1]

sin (C – D) = sin C cos D – cos C sin D [2]

[1] – [2] $\Rightarrow$ sin(C + D) – sin(C – D) = 2 cos C sin D

Let $C = \frac{A+B}{2}, D = \frac{A-B}{2}$

$\Rightarrow \sin\left(\frac{A+B}{2} + \frac{A-B}{2}\right) - \sin\left(\frac{A+B}{2} - \left(\frac{A-B}{2}\right)\right) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

(b) cos(C + D) = cos C cos D – sin D sin C [1]

cos(C – D) = cos C cos D + sin C sin D [2]

[1] + [2] ⇒ cos(C + D) + cos (C – D) = 2 cos C cos D

Let $C = \frac{A+B}{2}$, $D = \frac{A-B}{2}$

$$\Rightarrow \cos\left(\frac{A+B}{2} + \frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2} - \left(\frac{A-B}{2}\right)\right) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\Rightarrow \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

(c) cos(C + D) = cos C cos D – sin C sin D [1]

cos(C – D) = cos C cos D + sin C sin D [2]

[1] – [2] ⇒ cos (C + D) – cos(C – D) = –2 sin C sin D

Let $C = \frac{A+B}{2}$, $D = \frac{A-B}{2}$

$$\Rightarrow \cos\left(\frac{A+B}{2} + \frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2} - \left(\frac{A-B}{2}\right)\right) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\Rightarrow \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Exercise 9E

1 sin 4 x – sin x

$$= 2\cos\left(\frac{4x+x}{2}\right)\sin\left(\frac{4x-x}{2}\right)$$

$$= 2\cos\left(\frac{5}{2}x\right)\sin\left(\frac{3}{2}x\right)$$

2 cos 3x + cos 2x

$$= 2\cos\left(\frac{3x+2x}{2}\right)\cos\left(\frac{3x-2x}{2}\right)$$

$$= 2\cos\left(\frac{5}{2}x\right)\cos\left(\frac{1}{2}x\right)$$

3 $\cos 5A - \cos A = -2\sin\left(\frac{5A+A}{2}\right)\sin\left(\frac{5A-A}{2}\right)$

= –2 sin 3A sin 2A

4 $\sin 4A + \sin 4B = 2\sin\left(\frac{4A+4B}{2}\right)\cos\left(\frac{4A-4B}{2}\right)$

= 2 sin 2(A + B) cos 2(A – B)

5 $\cos 6A - \cos 4A = -2\sin\left(\frac{6A+4A}{2}\right)\sin\left(\frac{6A-4A}{2}\right)$

= – 2 sin 5A sin A

6 $\cos 2A - \cos 8A = -2\sin\left(\frac{2A+8A}{2}\right)\sin\left(\frac{2A-8A}{2}\right)$

= – 2 sin 5A sin (–3A)

= 2 sin 5A sin 3A

7 $\sin 7x + \sin 3x = 2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)$

$= 2\sin 5x \cos 2x$

8 $\cos 5x + \cos 3x = 2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)$

$= 2\cos 4x \cos x$

9 $\sin 6x - \sin 2x = 2\cos\left(\frac{6x+2x}{2}\right)\sin\left(\frac{6x-2x}{2}\right)$

$= 2\cos 4x \sin 2x$

10 $\sin 7x + \sin 5x = 2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)$

$= 2\sin 6x \cos x$

11 (a) $\cos\frac{5\pi}{12} + \cos\frac{\pi}{12}$

$= 2\cos\left(\frac{\frac{5\pi}{12}+\frac{\pi}{12}}{2}\right)\cos\left(\frac{\frac{5\pi}{12}-\frac{\pi}{12}}{2}\right)$

$= 2\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)$

$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{6}}{2}$

(b) $\cos\frac{5\pi}{12} - \cos\frac{\pi}{12}$

$= -2\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= -2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{-\sqrt{2}}{2}$

(c) $\sin\left(\frac{5\pi}{12}\right) - \sin\left(\frac{\pi}{12}\right)$

$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= 2\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{2}}{2}$

12 $\frac{\cos\alpha + \cos\beta}{\sin\alpha + \sin\beta}$

$$= \frac{\not{2}\cos\left(\frac{\alpha+\beta}{2}\right)\cancel{\cos\left(\frac{\alpha-\beta}{2}\right)}}{\not{2}\sin\left(\frac{\alpha+\beta}{2}\right)\cancel{\cos\left(\frac{\alpha-\beta}{2}\right)}}$$

$$= \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)} = \cot\left(\frac{\alpha+\beta}{2}\right)$$

13 $\sin 40 + \cos 70$

$= \cos 50 + \cos 70$

$= 2\cos\frac{50+70}{2}\cos\frac{50-70}{2}$

$= 2\cos 60\cos(-10)$

$= 2\cos 60\cos 10$

14 $\cos 5x + \cos x = 0$

$\Rightarrow 2\cos 3x\cos 2x = 0$

$\therefore \cos 3x = 0, \cos 2x = 0$

$3x = 2n\pi \pm \frac{\pi}{2}$ OR $2x = 2n\pi \pm \frac{\pi}{2}$

$x = \frac{2n\pi}{3} \pm \frac{\pi}{6}$ or $x = n\pi \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$

15 $\sin 6x + \sin 2x = 0$

$2\sin 4x\cos 2x = 0$

$\sin 4x = 0, \cos 2x = 0$

$4x = n\pi,\ 2x = 2n\pi \pm \frac{\pi}{2}$

$x = \frac{n\pi}{4}$ or $x = n\pi \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$

16 $\cos 6x - \cos 4x = 0$

$\Rightarrow -2\sin 5x\sin x = 0$

$\sin 5x = 0, \sin x = 0$

$5x = n\pi + (-1)^n$ or $x = n\pi + (-1)^n (0)$

$x = \frac{n\pi}{5}$ or $x = n\pi$, $n \in \mathbb{Z}$

17 $\sin 3x = \sin x$

$\Rightarrow \sin 3x - \sin x = 0$

$2\cos 2x\sin x = 0$

$\cos 2x = 0, \sin x = 0$

$2x = 2n\pi \pm \frac{\pi}{2},\ x = n\pi$

$$\left.\begin{aligned} \therefore x &= n\pi + \frac{\pi}{4} \\ x &= n\pi \end{aligned}\right\} n \in \mathbb{Z}$$

18 $\cos 6x + \cos 2x = \cos 4x$

$\Rightarrow 2\cos 4x\cos 2x = \cos 4x$

$\Rightarrow 2\cos 4x\cos 2x - \cos 4x = 0$

$\Rightarrow \cos 4x(2\cos 2x - 1) = 0$

cos 4x = 0, 2 cos 2x – 1 = 0

$\cos 2x = \frac{1}{2}$

$4x = 2n\pi \pm \frac{\pi}{2} \quad 2x = 2n\pi \pm \frac{\pi}{3}$

$x = \frac{2n\pi}{4} \pm \frac{\pi}{8}, \quad x = \frac{2n\pi}{2} \pm \frac{\pi}{6}$

$\left.\begin{array}{l} x = \frac{n\pi}{2} + \frac{\pi}{8} \\ \quad n\pi \pm \frac{\pi}{6} \end{array}\right\} n \in \square$

19 sin 7x + sin x = sin 4x

⇒ sin 7x + sin x – sin 4x = 0

⇒ 2 sin 4x cos 3x – sin 4x = 0

sin 4x [2 cos 3x – 1] = 0

$\sin 4x = 0, \quad \cos 3x = \frac{1}{2}$

$4x = n\pi, \quad 3x = 2n\pi \pm \frac{\pi}{3}$

$x = \frac{n\pi}{4}, \quad x = \frac{2n\pi}{3} \pm \frac{\pi}{9}, \quad n \in \square$

20 cos 5x – sin 3x – cos x = 0

⇒ cos 5x – cos x – sin 3x = 0

⇒ –2 sin 3x sin 2x – sin 3x = 0

⇒ –sin 3x [2sin 2x + 1] = 0

$\sin 3x = 0, \sin 2x = -\frac{1}{2}$

$3x = n\pi, \quad 2x = n\pi + (-1)^n \left(\frac{-\pi}{6}\right)$

$x = \frac{n\pi}{3}, \quad x = \frac{n\pi}{2} + (-1)^n \left(\frac{-\pi}{12}\right), n \in \square$

21 sin 3x + sin 4x + sin 5x = 0

sin 5x + sin 3x + sin 4x = 0

2 sin 4x cos x + sin 4x = 0

sin 4x (2 cos x + 1) = 0

$\sin 4x = 0, \ \cos x = -\frac{1}{2}$

$4x = n\pi, \quad x = 2n\pi \pm \frac{2\pi}{3}$

$x = \frac{n\pi}{4} \text{ or } x = 2n\pi \pm \frac{2\pi}{3}, \quad n \in \square$

22 sin x + 2 sin 2x + sin 3x = 0

sin 3x + sin x + 2 sin 2x = 0

2 sin 2x cos x + 2 sin 2x = 0

2 sin 2x (cos x + 1) = 0

sin 2x = 0, cos x + 1 = 0

2x = nπ, cos x = –1

x = 2nπ ± π

$x = \frac{n\pi}{2}$ or $x = 2n\pi \pm \pi, n \in \mathbb{Z}$

23 $\cos 3x + \cos x + 2\cos 2x = 0$

$2\cos 2x \cos x + 2\cos 2x = 0$

$2\cos 2x(\cos x + 1) = 0$

$\cos 2x = 0, \quad \cos x = -1$

$2x = 2n\pi \pm \frac{\pi}{2}, \quad x = 2n\pi \pm \pi$

$x = n\pi \pm \frac{\pi}{4}, x = 2n\pi \pm \pi, n \in \mathbb{Z}$

24 $\frac{\sin 4\theta + \sin\theta}{\cos 4\theta + \cos\theta}$

$= \frac{\cancel{2}\sin\left(\frac{5\theta}{2}\right)\cancel{\cos\left(\frac{3\theta}{2}\right)}}{\cancel{2}\cos\left(\frac{5\theta}{2}\right)\cancel{\cos\left(\frac{3\theta}{2}\right)}}$

$= \tan\left(\frac{5\theta}{2}\right)$

25 $\frac{\sin 6\theta - \sin 2\theta}{\cos 6\theta + \cos 2\theta}$

$= \frac{2\cancel{\cos 4\theta}\sin 2\theta}{2\cancel{\cos 4\theta}\cos 2\theta}$

$= \tan 2\theta$

26 $\frac{\sin 8\theta + \sin 4\theta}{\cos 8\theta - \cos 4\theta}$

$= \frac{2\cancel{\sin 6\theta}\cos 2\theta}{-2\cancel{\sin 6\theta}\sin 2\theta}$

$= -\cot 2\theta$

27 $\frac{\cos 7\theta + \cos\theta}{\sin 7\theta + \sin\theta}$

$= \frac{\cancel{2}\cos 4\theta\cos 3\theta}{\cancel{2}\sin 4\theta\cos 3\theta}$

$= \cot 4\theta$

28 $\frac{\sin x + 2\sin 3x + \sin 5x}{\sin 3x + 2\sin 5x + \sin 7x}$

$= \frac{\sin 5x + \sin x + 2\sin 3x}{\sin 7x + \sin 3x + 2\sin 5x}$

$= \frac{2\sin 3x\cos 2x + 2\sin 3x}{2\sin 5x\cos 2x + 2\sin 5x}$

$= \frac{\cancel{2}\sin 3x\cancel{(\cos 2x + 1)}}{\cancel{2}\sin 5x\cancel{(\cos 2x + 1)}} = \frac{\sin 3x}{\sin 5x}$

29 $$\frac{\sin x + \sin 2x}{\cos x - \cos 2x} = \frac{2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)}{-2\sin\left(\frac{3x}{2}\right)\sin\left(\frac{-x}{2}\right)}$$

$$= \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}$$

$$= \cot\left(\frac{x}{2}\right)$$

30 $$\frac{\sin 3x + \sin 2x}{\sin 3x - \sin 2x} = \frac{2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos\left(\frac{5x}{2}\right)\sin\left(\frac{x}{2}\right)} = \tan\left(\frac{5x}{2}\right)\cot\left(\frac{x}{2}\right)$$

31 $$\frac{\cos 3x + \cos x}{\sin 3x + \sin x} = \frac{2\cos 2x\cos x}{2\sin 2x\cos x}$$

$= \cot 2x$

32 $$\frac{\sin 7\theta + \sin\theta}{\cos 7\theta + \cos\theta} = \frac{2\sin 4\theta\cos 3\theta}{2\cos 4\theta\cos 3\theta}$$

$= \tan 4\theta$

33 $$\frac{\cos\theta + 2\cos 2\theta + \cos 3\theta}{\cos\theta - 2\cos 2\theta + \cos 3\theta}$$

$$= \frac{\cos 3\theta + \cos\theta + 2\cos 2\theta}{\cos 3\theta + \cos\theta - 2\cos 2\theta}$$

$$= \frac{2\cos 2\theta\cos\theta + 2\cos 2\theta}{2\cos 2\theta\cos\theta - 2\cos 2\theta}$$

$$= \frac{2\cos 2\theta(\cos\theta + 1)}{2\cos 2\theta(\cos\theta - 1)}$$

$$= \frac{\cos\theta + 1}{\cos\theta - 1}$$

$$= \frac{2\cos^2\frac{\theta}{2}}{-2\sin^2\frac{\theta}{2}} = -\cot^2\left(\frac{\theta}{2}\right)$$

34 $$\frac{\sin 4\theta + \sin 6\theta + \sin 5\theta}{\cos 4\theta + \cos 6\theta + \cos 5\theta}$$

$$= \frac{\sin 6\theta + \sin 4\theta + \sin 5\theta}{\cos 6\theta + \cos 4\theta + \cos 5\theta}$$

$$= \frac{2\sin 5\theta\cos\theta + \sin 5\theta}{2\cos 5\theta\cos\theta + \cos 5\theta}$$

$$= \frac{\sin 5\theta\,[2\cos\theta + 1]}{\cos 5\theta\,[2\cos\theta + 1]}$$

$$= \frac{\sin 5\theta}{\cos 5\theta}$$
$$= \tan 5\theta$$

35 $$\frac{\sin 6\theta + \sin 7\theta + \sin\theta + \sin 2\theta}{\cos 2\theta + \cos\theta + \cos 6\theta + \cos 7\theta}$$

$$= \frac{\sin 7\theta + \sin 2\theta + \sin 6\theta + \sin\theta}{\cos 7\theta + \cos 2\theta + \cos 6\theta + \cos\theta}$$

$$= \frac{2\sin\left(\frac{9\theta}{2}\right)\cos\left(\frac{5\theta}{2}\right) + 2\sin\left(\frac{7\theta}{2}\right)\cos\left(\frac{5\theta}{2}\right)}{2\cos\left(\frac{9\theta}{2}\right)\cos\left(\frac{5\theta}{2}\right) + 2\cos\left(\frac{7\theta}{2}\right)\cos\left(\frac{5\theta}{2}\right)}$$

$$= \frac{\cancel{2\cos\left(\frac{5\theta}{2}\right)}\left[\sin\left(\frac{9\theta}{2}\right) + \sin\left(\frac{7\theta}{2}\right)\right]}{\cancel{2\cos\left(\frac{5\theta}{2}\right)}\left[\cos\left(\frac{9\theta}{2}\right) + \cos\left(\frac{7\theta}{2}\right)\right]}$$

$$= \frac{\sin\left(\frac{9\theta}{2}\right) + \sin\left(\frac{7\theta}{2}\right)}{\cos\left(\frac{9\theta}{2}\right) + \cos\left(\frac{7\theta}{2}\right)}$$

$$= \frac{2\sin 4\theta\cos\left(\frac{\theta}{2}\right)}{2\cos 4\theta\cos\left(\frac{\theta}{2}\right)}$$

$$= \frac{\sin 4\theta}{\cos 4\theta}$$
$$= \tan 4\theta$$

36 $$\frac{\sin\theta + \sin 3\theta + \cos 5\theta + \cos 7\theta}{\sin 4\theta + \cos 8\theta + \cos 4\theta}$$

$$= \frac{2\sin 2\theta\cos\theta + 2\cos 6\theta\cos\theta}{2\sin 2\theta\cos 2\theta + 2\cos 6\theta\cos 2\theta}$$

$$= \frac{\cancel{2}\cos\theta\,\cancel{[\sin 2\theta + \cos 6\theta]}}{\cancel{2}\cos 2\theta\,\cancel{[\sin 2\theta + \cos 6\theta]}}$$

$$= \frac{\cos\theta}{2\cos^2\theta - 1}$$

$$= \frac{\frac{\cos\theta}{\cos^2\theta}}{2 - \frac{1}{\cos^2\theta}} = \frac{\sec\theta}{2 - \sec^2\theta}$$

37 $$\frac{\cos\theta - 2\cos 3\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 7\theta}$$

$$= \frac{2\cos 4\theta\cos 3\theta - 2\cos 3\theta}{2\cos 4\theta\cos 3\theta + 2\cos 3\theta}$$

$$= \frac{2\cos 3\theta\,[\cos 4\theta - 1]}{2\cos 3\theta\,[\cos 4\theta + 1]}$$

$$= \frac{\cos 4\theta - 1}{\cos 4\theta + 1}$$

$$= \frac{-2\sin^2 2\theta}{2\cos^2 2\theta}$$

$= -\tan^2(2\theta)$

38 $$\frac{\cos 5\theta + 2\cos 7\theta + \cos 9\theta}{\cos 5\theta - 2\cos 7\theta + \cos 9\theta}$$

$$= \frac{2\cos 7\theta\cos 2\theta + 2\cos 7\theta}{2\cos 7\theta\cos 2\theta - 2\cos 7\theta}$$

$$= \frac{2\cos 7\theta\,[\cos 2\theta + 1]}{2\cos 7\theta\,[\cos 2\theta - 1]} = \frac{\cos 2\theta + 1}{\cos 2\theta - 1} = \frac{2\cos^2\theta}{-2\sin^2\theta} = -\cot^2\theta$$

39 $2\sin 6\theta\cos\theta = \sin 7\theta + \sin 5\theta$

40 $-2\sin 8\theta\cos 4\theta = -[2\sin 8\theta\cos 4\theta]$

$= -[\sin 12\theta + \sin 4\theta]$

41 $2\cos 6\theta\cos 2\theta = \cos 8\theta + \cos 4\theta$

42 $2\sin 7\theta\sin\theta = -[-2\sin 7\theta\sin\theta]$

$= -[\cos 8\theta - \cos 6\theta]$

$= \cos 6\theta - \cos 8\theta$

43 $2\cos 7\theta\cos 3\theta = \cos 10\theta + \cos 4\theta$

44 $-2\sin 7\theta\cos 3\theta = -[2\sin 7\theta\cos 3\theta]$

$= -[\sin 10\theta + \sin 4\theta]$

$= -\sin 10\theta - \sin 4\theta$

45 RTP: $2\cos x(\sin 3x - \sin x) = \sin 4x$

Proof:

$2\cos x(\sin 3x - \sin x)$

$= 2\cos x\,[2\cos 2x\sin x]$

$= 2\cos 2x\,[2\sin x\cos x]$

$= 2\cos 2x\sin 2x$

$= \sin 4x$

46 $$\sin 5x + \sin x = 2\sin\left(\frac{5x + x}{2}\right)\cos\left(\frac{5x - x}{2}\right)$$

$= 2\sin 3x\cos 2x.$

$\sin 5x + \sin x + \cos 2x = 0$

$\Rightarrow 2\sin 3x\cos 2x + \cos 2x = 0$

$\cos 2x\,[2\sin 3x + 1] = 0$

$$\cos 2x = 0, \quad \sin 3x = -\frac{1}{2}$$

$$2x = 2n\pi \pm \frac{\pi}{2}, \quad 3x = n\pi + (-1)^n\left(\frac{-\pi}{6}\right)$$

$$\left.\begin{aligned} x &= n\pi \pm \frac{\pi}{4} \\ x &= \frac{n\pi}{3} + (-1)^n\left(\frac{-\pi}{18}\right)\end{aligned}\right\} n \in \square$$

47 (a) RTP: $\sin 2P + \sin 2Q + \sin 2R = 4\sin P\sin Q\sin R$ where $P + Q + R = 180°$

Proof:

$\sin 2P + \sin 2Q + \sin 2R$

$= \sin 2P + 2\sin(Q + R)\cos(Q - R)$

$= \sin 2P + 2\sin(180 - P)\cos(Q - R)$, [$Q + R = 180 - P$]

$= 2\sin P\cos P + 2\sin P\cos(Q - R)$, [$\sin(180 - P) = \sin P$]

$= 2\sin P[\cos P + \cos(Q - R)]$

$= 2\sin P\left[2\cos\frac{P+Q-R}{2}\cos\left(\frac{P-Q+R}{2}\right)\right]$

$= 2\sin P[2\cos(90 - R)\cos(90 - Q)]$ [$P + Q - R = 180 - 2R$, $\frac{P+Q-R}{2} = 90 - R$, $P + R - Q = 180 - 2Q$, $\frac{P+R-Q}{2} = 90 - Q$]

$= 4\sin P\sin Q\sin R$ [$\cos(90 - R) = \sin R$, $\cos(90 - Q) = \sin Q$]

(b) $\sin 2P + \sin 2Q - \sin 2R$

$= 2\sin P\cos P + 2\cos(Q + R)\sin(Q - R)$

$= 2\sin P\cos P + 2\cos(180 - P)\sin(Q - R)$

$= 2\sin P\cos P - 2\cos P\sin(Q - R)$.

$= 2\cos P[\sin P - \sin(Q - R)]$

$= 2\cos P\left[2\cos\frac{P+Q-R}{2}\sin\frac{P+R-Q}{2}\right]$

$= 2\cos P[2\cos(90 - R)\sin(90 - Q)]$

$= 4\cos P\cos Q\sin R$

48 (a) Proof:

$P + Q + R = 180°$

$\sin(Q + R) = \sin(180 - P)$

$= \sin P$

(b) $\cos(Q + R) = \cos(180 - P)$

$= -\cos P$

49 (a) $\alpha + \beta + \gamma = 180°$.

$\sin\beta\cos\gamma + \cos\beta\sin\gamma$

$= \sin(\beta + \gamma)$

$= \sin(180 - \alpha)$

$= \sin\alpha$

(b) $\cos\gamma + \cos\beta\cos\alpha$

$= \cos(180 - (\beta + \alpha)) + \cos\beta\cos\alpha$

$= -\cos(\beta + \alpha) + \cos\beta\cos\alpha$

$= -\cancel{\cos\beta\cos\alpha} + \sin\beta\sin\alpha + \cancel{\cos\beta\cos\alpha}$

$= \sin\beta\sin\alpha$

(c) $\sin\alpha - \cos\beta\sin\gamma$

$= \sin(180 - \alpha) - \cos\beta\sin\gamma$

$= \sin\beta\cos\gamma + \cancel{\cos\beta\sin\gamma} - \cancel{\cos\beta\sin\gamma}$

$= \sin\beta\cos\gamma$

50 $1 + \cos 2\theta + \cos 4\theta + \cos 6\theta$

$= 1 + \cos 2\theta + 2\cos 5\theta\cos\theta$

$= 2\cos^2\theta + 2\cos 5\theta \cos\theta$

$= 2\cos\theta\,[\cos\theta + \cos 5\theta]$

$= 2\cos\theta\,[2\cos 3\theta \;\; \cos 2\theta]$

$= 4\cos\theta\cos 2\theta\cos 3\theta$

Now $1 + \cos 2\theta + \cos 4\theta + \cos 6\theta = 0$

$\Rightarrow 4\cos\theta\cos 2\theta\cos 3\theta = 0.$

$\Rightarrow \cos\theta = 0, \cos 2\theta = 0, \cos 3\theta = 0$

$$\theta = 2n\pi \pm \frac{\pi}{2}, \quad 2\theta = 2n\pi \pm \frac{\pi}{2}, \quad 3\theta = 2n\pi \pm \frac{\pi}{2}$$

$$\left.\begin{array}{l} \theta = 2n\pi \pm \dfrac{\pi}{2} \\ \theta = n\pi \pm \dfrac{\pi}{4} \\ \theta = \dfrac{2n\pi}{3} \pm \dfrac{\pi}{6} \end{array}\right\} n \in \mathbb{Z}$$

51 $1 - \cos 2\theta + \cos 4\theta - \cos 6\theta$

$= 2\sin^2\theta + [-2\sin(-\theta)\sin 5\theta]$

$= 2\sin^2\theta + 2\sin\theta\sin 5\theta$

$= 2\sin\theta\,[\sin\theta + \sin 5\theta]$

$= 2\sin\theta\,[2\cos 2\theta\sin 3\theta]$

$= 4\sin\theta\cos 2\theta\sin 3\theta$

Now $1 - \cos 2\theta + \cos 4\theta - \cos 6\theta = 0$

$\Rightarrow 4\sin\theta\cos 2\theta\sin 3\theta = 0$

$\Rightarrow \sin\theta = 0, \cos 2\theta = 0, \sin 3\theta = 0$

$$\theta = n\pi, \quad 2\theta = 2n\pi \pm \frac{\pi}{2}, \quad 3\theta = n\pi$$

$$\left.\begin{array}{l} \theta = n\pi \\ \quad n\pi \pm \dfrac{\pi}{4} \\ \quad \dfrac{n\pi}{3} \end{array}\right\} n \in \mathbb{Z}$$

Review Exercise 9

1 (a) RTP: $\dfrac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

Proof:

$$\frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$= \sec\theta + \tan\theta$

(b) $$\frac{1}{\sec x - \tan x} = \frac{1}{\sec x - \tan x} \times \frac{\sec x + \tan x}{\sec x + \tan x}$$

$$= \frac{\sec x + \tan x}{\sec^2 x - \tan^2 x}$$

$= \sec x + \tan x$ [since $\sec^2 x - \tan^2 x = 1$]

2 (a) $3\cos^2 x = 1 + \sin x$

$\Rightarrow 3(1 - \sin^2 x) = 1 + \sin x$

$\Rightarrow 3 - 3\sin^2 x = 1 + \sin x$

$\Rightarrow 3\sin^2 x + \sin x - 2 = 0.$

$y = \sin x$

$3y^2 + y - 2 = 0$

$(3y - 2)(y + 1) = 0$

$$y = \frac{2}{3}, \ -1$$

$$\sin x = \frac{2}{3}, \quad \sin x = -1$$

$$\left.\begin{aligned} x &= n\pi + (-1)^n(0.730) \\ x &= n\pi + (-1)^n\left(\frac{-\pi}{2}\right) \end{aligned}\right\} n \in \mathbb{Z}$$

(b) $3\cos x = 2\sin^2 x$

$3\cos x = 2(1 - \cos^2 x)$

$2\cos^2 x + 3\cos x - 2 = 0$

$y = \cos x$

$2y^2 + 3y - 2 = 0$

$(2y - 1)(y + 2) = 0$

$$y = \frac{1}{2}, -2$$

$$\cos x = \frac{1}{2}, \quad \cos x = -2 \Rightarrow \text{No solutions}$$

$$x = 2n\pi \pm \frac{\pi}{3}$$

3 $$\sqrt{\frac{9 + 9\cos 6\alpha}{2}} = \sqrt{\frac{9(1 + \cos 6\alpha)}{2}}$$

$$= \sqrt{9\cos^2(3\alpha)}$$

$= 3\cos 3\alpha$

4 $$\frac{\cos 3\alpha}{\cos\alpha} + \frac{\sin 3\alpha}{\sin\alpha}$$

$$= \frac{\cos 3\alpha \sin\alpha + \sin 3\alpha \cos\alpha}{\cos\alpha \sin\alpha}$$

$$= \frac{\sin(\alpha + 3\alpha)}{\frac{1}{2}\sin 2\alpha}$$

$$=\frac{\sin 4\alpha}{\frac{1}{2}\sin 2\alpha}$$

$$=\frac{2\ \sin 2\alpha\ \cos 2\alpha}{\frac{1}{2}\ \sin 2\alpha}$$

$= 4\cos 2\alpha$

5 RTP: $\sin(\alpha+\beta)\sin(\alpha-\beta)=\sin^2\alpha-\sin^2\beta$

Proof:

$\sin(\alpha+\beta)\sin(\alpha-\beta)$

$= (\sin\alpha\cos\beta+\cos\alpha\sin\beta)(\sin\alpha\cos\beta-\cos\alpha\sin\beta)$

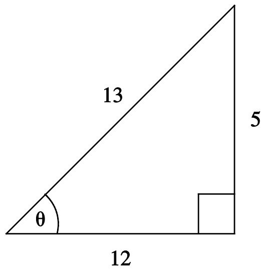
$= \sin^2\alpha\cos^2\beta-\cos^2\alpha\sin^2\beta$

$= \sin^2\alpha(1-\sin^2\beta)-\sin^2\beta(1-\sin^2\alpha)$

$= \sin^2\alpha-\sin^2\alpha\sin^2\beta-\sin^2\beta+\sin^2\beta\sin^2\alpha$

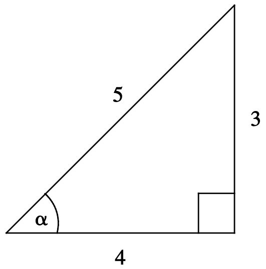
$= \sin^2\alpha-\sin^2\beta$

6 $\sin\theta=\frac{5}{13}$



$\cos\theta=\frac{12}{13}$

$\tan\theta=\frac{5}{12}$



$\sin\alpha=\frac{-3}{5}$

$\cos\alpha=\frac{-4}{5}$

$\tan\alpha=\frac{3}{4}$

(a) $\sin(\theta+\alpha)$

$= \sin\theta\cos\alpha+\cos\theta\sin\alpha$

$$=\left(\frac{5}{13}\right)\left(\frac{-4}{5}\right)+\left(\frac{12}{13}\right)\left(\frac{-3}{5}\right)$$

$= \frac{-20}{65} - \frac{36}{65}$

$= \frac{-56}{65}$

(b) $\cos(\theta + \alpha) = \cos\theta \cos\alpha - \sin\theta \sin\alpha$

$= \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) - \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right)$

$= \frac{-48}{65} + \frac{15}{65} = \frac{-33}{65}$

7 (a) RTP:

$\frac{\cos\theta - 2\cos 3\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 7\theta} = -\tan^2(2\theta)$

Proof:

$\frac{\cos\theta - 2\cos 3\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 7\theta}$

$= \frac{2\cos 4\theta \cos 3\theta - 2\cos 3\theta}{2\cos 4\theta \cos 3\theta + 2\cos 3\theta}$

$= \frac{\cancel{2\cos 3\theta}(\cos 4\theta - 1)}{\cancel{2\cos 3\theta}(\cos 4\theta + 1)}$

$= \frac{\cos 4\theta - 1}{\cos 4\theta + 1}$

$= \frac{\cancel{1} - 2\sin^2 2\theta - \cancel{1}}{2\cos^2 2\theta - \cancel{1} + \cancel{1}}$

$= \frac{-\cancel{2}\sin^2 2\theta}{\cancel{2}\cos^2 2\theta}$

$= \frac{-\sin^2 2\theta}{\cos^2 2\theta}$

$= -\tan^2(2\theta)$

(b) RTP:

$\frac{\cos 5\theta + 2\cos 7\theta + \cos 9\theta}{\cos 5\theta - 2\cos 7\theta + \cos 9\theta} = -\cot^2\theta$

Proof:

$\frac{\cos 5\theta + 2\cos 7\theta + \cos 9\theta}{\cos 5\theta - 2\cos 7\theta + \cos 9\theta}$

$= \frac{2\cos 7\theta \cos 2\theta + 2\cos 7\theta}{2\cos 7\theta \cos 2\theta - 2\cos 7\theta}$

$= \frac{\cancel{2\cos 7\theta}[\cos 2\theta + 1]}{\cancel{2\cos 7\theta}[\cos 2\theta - 1]}$

$= \frac{2\cos^2\theta - \cancel{1} + \cancel{1}}{\cancel{1} - 2\sin^2\theta - \cancel{1}}$

$= \frac{\cancel{2}\cos^2\theta}{-\cancel{2}\sin^2\theta}$

$= -\cot^2\theta$

8 cos 15° = cos(60° – 45°)

= cos 60 cos 45 – sin 60 sin 45

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{4}[1-\sqrt{3}]$

9 $\frac{\sin(\theta-\alpha)}{\sin\theta\sin\alpha} = \frac{\sin\theta\cos\alpha - \cos\theta\sin\alpha}{\sin\theta\sin\alpha}$

$= \frac{\sin\theta\cos\alpha}{\sin\theta\sin\alpha} - \frac{\cos\theta\sin\alpha}{\sin\theta\sin\alpha}$

$= \frac{\cos\alpha}{\sin\alpha} - \frac{\cos\theta}{\sin\theta}$

= cot α – cot θ

10 $\sin\left(\frac{\pi}{4}+x\right)\cos\left(\frac{\pi}{4}+x\right)$

$= \frac{1}{2}\sin\left(\frac{\pi}{2}+2x\right)$

$= \frac{1}{2}\cos 2x$

Since $\sin(90+\alpha) = \cos\alpha$

11 f(θ) = 3 sin θ + 4 cos θ

3 sin θ + 4 cos θ = r sin (θ + α)

= r sin θ cos α + r cos θ sin α

Comparing coefficients of sin θ and cos θ

⇒ r cos α = 3 [1]

r sin α = 4 [2]

$[2]\div[1] \Rightarrow \frac{r\sin\alpha}{r\cos\alpha} = \frac{4}{3} \Rightarrow \tan\alpha = \frac{4}{3}, \alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°$

$[1]^2 + [2]^2 \Rightarrow r^2\sin^2\alpha + r^2\cos^2\alpha = 4^2 + 3^2$

$r^2 = 25$

r = 5

∴ f(θ) = 5 sin (θ + 53.1°)

max f(θ) = 5

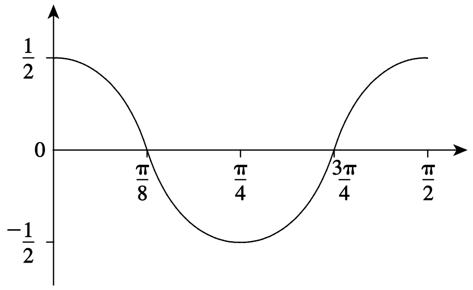
$\min\left[\frac{1}{10+f(\theta)}\right] = \frac{1}{10+5} = \frac{1}{15}$

12 $f(x) = \frac{1}{2}\sin\left(4x+\frac{\pi}{2}\right)$

(a) Range: $\frac{-1}{2} \le f(x) \le \frac{1}{2}$

(b) Period: $\frac{\pi}{2}$

(c)



13 (a) $2\cos(2x) + 3\sin 2x = r\cos(2x - \theta)$

$= r\cos 2x\cos\theta + r\sin 2x\sin\theta$

Equating coefficients of cos 2x and sin 2x

$\Rightarrow r\cos\theta = 2$ [1]

$r\sin\theta = 3$ [2]

$[2] \div [1] \Rightarrow \dfrac{r\sin\theta}{r\cos\theta} = \dfrac{3}{2}$

$\tan\theta = \dfrac{3}{2} \Rightarrow \theta = \tan^{-1}\left(\dfrac{3}{2}\right) = 56.3°$

$[1]^2 + [2]^2 \Rightarrow r^2\cos^2\theta + r^2\sin^2\theta = 2^2 + 3^2$

$r^2 = 13$

$r = \sqrt{13}$

$\therefore\ 3\cos 2x + 3\sin 2x = \sqrt{13}\cos(2x - 56.3°)$

(b) $\sqrt{13}\cos(2x - 56.3°) = 2$

$\cos(2x - 56.3°) = \dfrac{2}{\sqrt{13}}$

$2x - 56.3 = \cos^{-1}\left(\dfrac{2}{\sqrt{13}}\right)$

$\Rightarrow 2x - 56.3 = 360n \pm 56.3°$

$2x = 360n + 112.6°, 360°n$

$x = 180°n + 56.3°, 180°n, n \in \mathbb{Z}$

(c) Maximum value $= \sqrt{13}$

$\cos(2x - 56.3) = 1$

$2x - 56.3 = 0$

$x = 28.2°$

14 $2\sin 6\theta\cos\theta = \sin 7\theta + \sin 5\theta$

15 $-2\sin 8\theta\cos 4\theta = -[\sin 12\theta + \sin 4\theta]$

$= -\sin 12\theta - \sin 4\theta$

16 $2\cos 6\theta\cos 2\theta = \cos 8\theta + \cos 4\theta$

17 (a) $2\tan x - 1 = 3\cot x$

$2\tan x - 1 = \dfrac{3}{\tan x}$

$\Rightarrow 2\tan^2 x - \tan x - 3 = 0$

$(2\tan x - 3)(\tan x + 1) = 0$

$\tan x = \dfrac{3}{2}, \quad \tan x = -1$

$$\left.\begin{array}{l} x = \pi + 0.983 \\ x = \pi \;\; -\dfrac{\pi}{4} \end{array}\right\} n \in \square$$

(b) $6 \sec^2 z = \tan z + 8$

$6\,(1 + \tan^2 z) = \tan z + 8$

$6 \tan^2 z - \tan z - 2 = 0$

$(2 \tan z + 1)\,(3 \tan z - 2) = 0$

$\tan z = \dfrac{-1}{2} \quad \tan z = \dfrac{2}{3}$

$$\left.\begin{array}{l} z = \pi \;\; -0.464 \\ z = \pi + 0.588 \end{array}\right\} n \in \square$$

18 (a) RTP: $\dfrac{\sin x}{1 - \sec x} + \dfrac{\sin x}{1 + \sec x} = -2 \cos x \cot x$

Proof:

$$\frac{\sin x}{1 - \sec x} + \frac{\sin x}{1 + \sec x}$$

$$= \frac{\sin x(1 + \sec x) + \sin x(1 - \sec x)}{(1 - \sec x)\,(1 + \sec x)}$$

$$= \frac{\sin x + \cancel{\sin x \sec x} + \sin x - \cancel{\sin x \sec x}}{1 - \sec^2 x}$$

$$= \frac{2 \sin x}{\cancel{1} - \cancel{1} - \tan^2 x}$$

$$= \frac{2 \sin x}{\dfrac{-\sin^2 x}{\cos^2 x}} = \frac{-2 \cos^2 x}{\sin x}$$

$= -2 \cot x \cos x$

(b) RTP: $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$

Proof:

$$\frac{1 - \sin x}{\cos x} = \frac{1 - \sin x}{\cos x} \times \frac{1 + \sin x}{1 + \sin x}$$

$$= \frac{1 - \sin^2 x}{\cos x(1 + \sin x)}$$

$$= \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}\,(1 + \sin x)}$$

$$= \frac{\cos x}{1 + \sin x}$$

OR:

$$\frac{1 - \sin x}{\cos x} = \frac{\cos^2 \dfrac{x}{2} + \sin^2 \dfrac{x}{2} - 2 \sin \dfrac{x}{2} \cos \dfrac{x}{2}}{\cos^2 \dfrac{x}{2} - \sin^2 \left(\dfrac{x}{2}\right)}$$

$$= \frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^{\not 2}}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} \times \frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

$$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} + \sin^2\frac{x}{2}}$$

$$= \frac{\cos x}{1 + \sin x}$$

(c) $\cot\theta + \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

$= \sec\theta \operatorname{cosec}\theta$

19 (a) $\cos 5x - \sin 3x - \cos x = 0$

$\Rightarrow \cos 5x - \cos x - \sin 3x = 0$

$\Rightarrow -2\sin 3x \sin 2x - \sin 3x = 0$

$\Rightarrow -\sin 3x\,(2 \sin 2x + 1) = 0$

$$\sin 3x = 0, \quad \sin 2x = \frac{-1}{2}$$

$$3x = n\pi \Rightarrow x = \frac{n\pi}{3}$$

$$2x = n\pi + (-1)^n\left(\frac{-\pi}{6}\right)$$

$$\left.\begin{cases} x = \dfrac{n\pi}{2} + (-1)^n\left(\dfrac{-\pi}{12}\right) \\ x = \dfrac{n\pi}{3} \end{cases}\right\} n \in \mathbb{Z}$$

(b) $\sin 3x + \sin 4x + \sin 5x = 0$

$\sin 4x + \sin 3x + \sin 5x = 0$

$\sin 4x + 2 \sin 4x \cos x = 0$

$\sin 4x\,(1 + 2 \cos x) = 0$

$$\sin 4x = 0, \quad \cos x = \frac{-1}{2}$$

$4x = n\pi, \quad x = 2n\pi \pm \dfrac{2\pi}{3}$

$$\left.\begin{array}{l} x = \dfrac{n\pi}{4} \\ x = 2\boldsymbol{\pi} \pm \dfrac{\pi}{3} \end{array}\right\} n \in \square$$

20 (a) $\dfrac{\sin 7\theta - \sin\theta}{\cos 7\theta + \cos\theta}$

$$= \frac{\cancel{2\cos 4\theta}\ \sin 3\theta}{\cancel{2\cos 4\theta}\ \cos 3\theta}$$

$$= \frac{\sin 3\theta}{\cos 3\theta}$$

$= \tan 3\theta$

(b) $\dfrac{\cos\theta + 2\cos 2\theta + \cos 3\theta}{\cos\theta - 2\cos 2\theta + \cos 3\theta}$

$$= \frac{2\cos 2\theta\cos\theta + 2\cos 2\theta}{2\cos 2\theta\cos\theta - 2\cos 2\theta}$$

$$= \frac{\cancel{2\cos 2\theta}(\cos\theta + 1)}{\cancel{2\cos 2\theta}\ (\cos\theta - 1)}$$

$$= \frac{\cancel{2}\ \cos^2\left(\frac{\theta}{2}\right)}{-\cancel{2}\ \sin^2\left(\frac{\theta}{2}\right)}$$

$$= -\cot^2\left(\frac{\theta}{2}\right)$$