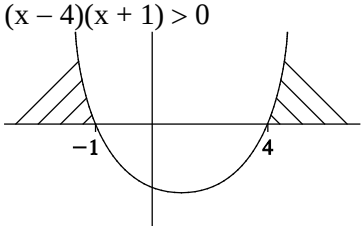
Chapter 8 Inequalities and the Modulus Function

Try these 8.1

(a) $3x < x^2 - 4$

$\Rightarrow x^2 - 3x - 4 > 0$

$(x - 4)(x + 1) > 0$

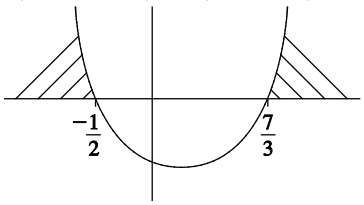


$\{x: x < -1\} \cup \{x: x > 4\}$

(b) $6x^2 - 11x - 7 \geq 0$

$(2x + 1)(3x - 7) \geq 0$

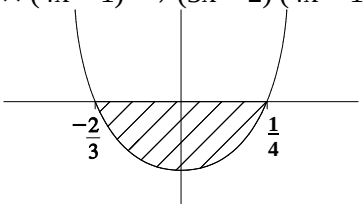
$\left\{x : x \leq -\frac{1}{2}\right\} \cup \left\{x : x \geq \frac{7}{3}\right\}$



Try these 8.2

(a) $\frac{3x + 2}{4x - 1} < 0$

$\times (4x - 1)^2 \Rightarrow (3x + 2)(4x - 1) < 0$



$\therefore \left\{x : \frac{-2}{3} < x < \frac{1}{4}\right\}$

(b) $\frac{3x - 4}{x - 2} < 5$

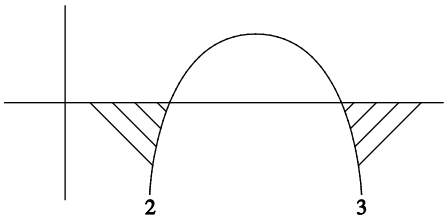
$\times (x - 2)^2 \Rightarrow (3x - 4)(x - 2) < 5(x - 2)^2$

$\Rightarrow (3x - 4)(x - 2) - 5(x - 2)^2 < 0$

$\Rightarrow (x - 2)[3x - 4 - 5x + 10] < 0$

$\Rightarrow (x - 2)(-2x + 6) < 0$

$\{x: x < 2\} \cup \{x: x > 3\}$



(c) $\frac{x+1}{x+2} > 3$

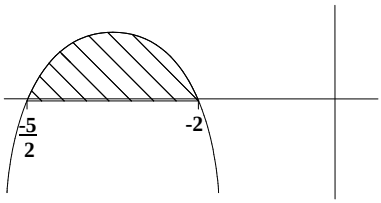
$\times (x+2)^2 \Rightarrow (x+1)(x+2) > 3(x+2)^2 -$

$\Rightarrow (x+1)(x+2) - 3(x+2)^2 > 0$

$\Rightarrow (x+2)[x+1-3(x+2)] > 0$

$\Rightarrow (x+2)(-2x-5) > 0$

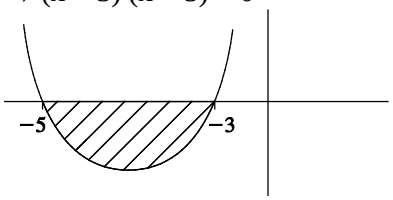
$\Rightarrow \left\{x : \frac{-5}{2} < x < -2\right\}$



Exercise 8A

1 $x^2 + 8x + 15 < 0$

$\Rightarrow (x+3)(x+5) < 0$

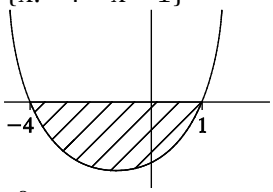


$\Rightarrow \{x: -5 < x < -3\}$

2 $x^2 + 3x - 4 < 0$

$\Rightarrow (x+4)(x-1) < 0$

$\{x: -4 < x < 1\}$

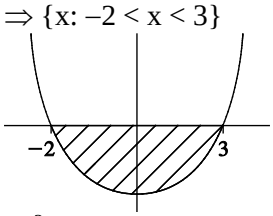


3 $x^2 - x < 6$

$\Rightarrow x^2 - x - 6 < 0$

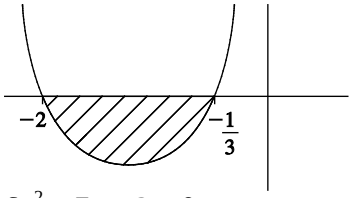
$\therefore (x-3)(x+2) < 0$

$\Rightarrow \{x: -2 < x < 3\}$

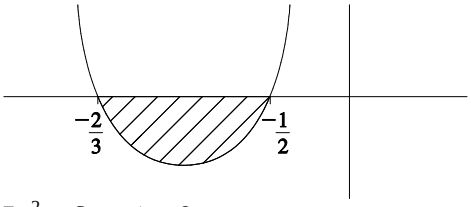


4 $3x^2 + 4x < -3x - 2$

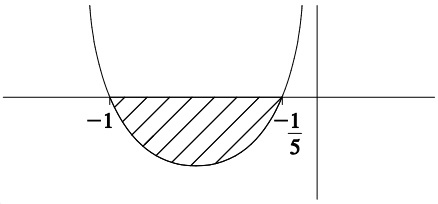
$\Rightarrow 3x^2 + 7x + 2 < 0$
$\Rightarrow (3x + 1)(x + 2) < 0$
$\therefore \left\{x : -2 < x < -\frac{1}{3}\right\}$



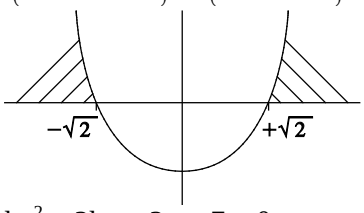
5 $6x^2 + 7x + 2 < 0$
$(3x + 2)(2x + 1) < 0$
$\left\{x : \frac{-2}{3} < x < \frac{-1}{2}\right\}$



6 $5x^2 + 6x + 1 < 0$
$(5x + 1)(x + 1) < 0$
$\left\{x : -1 < x < -\frac{1}{5}\right\}$



7 $x^2 - 2 > 0$
$(x - \sqrt{2})(x + \sqrt{2}) > 0$
$\left\{x : x < -\sqrt{2}\right\} \cup \left\{x : x > \sqrt{2}\right\}$



8 $kx^2 + 2kx + 2x + 7 = 0$
$kx^2 + x(2k + 2) + 7 = 0$
For real roots, $b^2 - 4ac \geq 0$
$\therefore (2k + 2)^2 - 4(k)(7) \geq 0$
$\Rightarrow 4k^2 + 8k + 4 - 28k \geq 0$
$4k^2 - 20k + 4 \geq 0$
$k^2 - 5k + 1 \geq 0$
$k = \frac{5 \pm \sqrt{21}}{2}$

$$\left(k - \frac{\left(5+\sqrt{21}\right)}{2}\right)\left(k - \frac{5-\sqrt{21}}{2}\right) \geq 0$$

$$\left\{k : k \leq \frac{5-\sqrt{21}}{2}\right\} \cup \left\{k : k \geq \frac{5+\sqrt{21}}{2}\right\}$$

9 $2x^2 + 5x + k + 2 = 0$
For real and distinct roots $b^2 - 4ac > 0$
$\Rightarrow (5)^2 - 4(2)(k + 2) > 0$
$\Rightarrow 25 - 8k - 16 > 0$
$8k < 9$
$k < \frac{9}{8}$

10 $(p + 2)x^2 - 4x + 3 = 0$
For real and distinct roots $b^2 - 4ac > 0$
$\Rightarrow (-4)^2 - 4(p + 2)(3) > 0$
$16 - 12p - 24 > 0$
$12p < -8$
$p < -8/12 = \frac{-2}{3}$

11 $4x^2 - 2kx + 3 = 0$
For real roots $b^2 - 4ac \geq 0$
$\Rightarrow (-2k)^2 - 4(4)(3) \geq 0$
$4k^2 - 48 \geq 0$
$k^2 - 12 \geq 0$
$k^2 \geq 12$
The smallest possible positive integer value for k is 4

12 (a) $\frac{x+4}{x+5} > 2$

$\Rightarrow \frac{x+4}{x+5} - 2 > 0$

$\Rightarrow \frac{x+4-2(x+5)}{x+5} > 0$

$\Rightarrow \frac{-x-6}{x+5} > 0$

Critical values: $-6, -5$

	$-x-6$	$x+5$	$\frac{-x-6}{x+5}$
$x < -6$	+ve	–ve	–ve
$-6 < x < -5$	–ve	–ve	+ve
$x > -5$	–ve	+ve	–ve

$\therefore \{x: -6 < x < -5\}$

(b) $\frac{2x-1}{3x+1} > 1$

$\Rightarrow \frac{2x-1}{3x+1} - 1 > 0$

$$\Rightarrow \frac{2x-1-3x-1}{3x+1} > 0$$

$$\Rightarrow \frac{-x-2}{3x+1} > 0$$

Critical values: –2, –1/3

	–x–2	3x+1	$\frac{-x-2}{3x+1}$
$x < -2$	+ve	–ve	–ve
$-2 < x < -\frac{1}{3}$	–ve	–ve	+ve
$x > -\frac{1}{3}$	–ve	+ve	–ve

$$\therefore \left\{x, -2 < x < -\frac{1}{3}\right\}$$

(c) $\frac{7x+2}{x+1} > 5$

$$\frac{7x+2}{x+1} - 5 > 0$$

$$\frac{7x+2-5x-5}{x+1} > 0$$

$$\frac{2x-3}{x+1} > 0$$

Critical values: $\frac{3}{2}$, –1

	2x–3	x+1	$\frac{2x-3}{x+1}$
$x < -1$	–ve	–ve	+ve
$-1 < x < \frac{3}{2}$	–ve	+ve	–ve
$x > \frac{3}{2}$	+ve	+ve	+ve

$\therefore \{x: x < -1\} \cup \{x: x > 2\}$

(d) $\frac{3x-1}{x-2} < 1$

$$\Rightarrow \frac{3x-1}{x-2} - 1 < 0$$

$$\Rightarrow \frac{3x-1-x+2}{x-2} < 0$$

$$\Rightarrow \frac{2x+1}{x-2} < 0$$

Critical values: 2, $-\frac{1}{2}$

	2x+1	x–2	$\frac{2x+1}{x-2}$
$x < -\frac{1}{2}$	–ve	–ve	+ve
$-\frac{1}{2} < x < 2$	+ve	–ve	–ve
$x > 2$	+ve	+ve	+ve

$\therefore \left\{x : -\frac{1}{2} < x < 2\right\}$

13 $\frac{x+1}{x-2} > \frac{x}{x+3}$

$\Rightarrow \frac{x+1}{x-2} - \frac{x}{x+3} > 0$

$\Rightarrow \frac{(x+1)(x+3) - x(x-2)}{(x-2)(x+3)} > 0$

$\Rightarrow \frac{\cancel{x^2} + 4x + 3 - \cancel{x^2} + 2x}{(x-2)(x+3)} > 0$

$\Rightarrow \frac{6x+3}{(x-2)(x+3)} > 0.$

Critical values are : –3, $-\frac{1}{2}$, 2

	6x+3	x–2	x+3	$\frac{6x+3}{(x-2)(x+3)}$
$x < -3$	–ve	–ve	–ve	–ve
$-3 < x < -\frac{1}{2}$	–ve	–ve	+ve	+ve
$-\frac{1}{2} < x < 2$	+ve	–ve	+ve	–ve
$x > 2$	+ve	+ve	+ve	+ve

$\therefore \quad \left\{x : -3 < x < -\frac{1}{2}\right\} \cup \{x : x > 2\}$

14 $\frac{2x+1}{x-3} < \frac{1}{x+2}$

$\Rightarrow \frac{2x+1}{x-3} - \frac{1}{x+2} < 0$

$\Rightarrow \frac{(2x+1)(x+2) - (x-3)}{(x-3)(x+2)} < 0$

$\Rightarrow \frac{2x^2 + 5x + 2 - x + 3}{(x-3)(x+2)} < 0$

$\Rightarrow \frac{2x^2 + 4x + 5}{(x-3)(x+2)} < 0$

Critical values –2, 3

	$2x^2+4x+5$	$x-3$	$x+2$	$\frac{2x^2+4x+5}{(x-3)(x+2)}$
$x < -2$	+ve	–ve	–ve	+ve
$-2 < x < 3$	+ve	–ve	+ve	–ve
$x > 3$	+ve	+ve	+ve	+ve

$\therefore \quad \{x: -2 < x < 3\}$

Try these 8.3

(a) $|x + 1| = 3$
$\Rightarrow x + 1 = 3, x + 1 = -3$
$x = 2, x = -4$
Hence $x = 2, -4$

(b) $|4x - 3| = 7$
$\Rightarrow 4x - 3 = 7, 4x - 3 = -7$
$4x = 10, 4x = -4$
$x = \frac{10}{4} = \frac{5}{2}, x = -1$
Hence $x = \frac{5}{2}, -1$

(c) $|2x + 5| = |4x - 7|$
$\Rightarrow |2x + 5|^2 = |4x - 7|^2$
$\Rightarrow (2x + 5)^2 = (4x - 7)^2$
$\Rightarrow 4x^2 + 20x + 25 = 16x^2 - 56x + 49$
$12x^2 - 76x + 24 = 0$
$3x^2 - 19x + 6 = 0$
$(3x - 1)(x - 6) = 0$
$x = \frac{1}{3}, 6$

Exercise 8B

1 (a) $|2x + 3| = 7$
$2x + 3 = 7, 2x + 3 = -7$
$2x = 4, \quad 2x = -10$
$x = 2, \quad x = -5$

(b) $|5x - 1| = 8$
$\Rightarrow 5x - 1 = 8, 5x - 1 = -8$
$5x = 9, 5x = -7$
$x = \frac{9}{5}, x = \frac{-7}{5}$

(c) $|4x + 3| = 1$
$\Rightarrow 4x + 3 = 1, 4x + 3 = -1$
$4x = -2, 4x = -4$
$x = \frac{-1}{2}, x = -1$

(d) $|1 - 2x| = 6$
$\Rightarrow 1 - 2x = 6, 1 - 2x = -6$
$2x = -5, 2x = 7$
$x = \frac{-5}{2}, x = \frac{7}{2}$

2 (a) $|3x + 1| = |2x - 4|$
squaring both sides $\Rightarrow |3x + 1|^2 = |2x - 4|^2$
$(3x + 1)^2 = (2x - 4)^2$
$9x^2 + 6x + 1 = 4x^2 - 16x + 16$
$5x^2 + 22x - 15 = 0$
$(5x - 3)(x + 5) = 0$
$x = \frac{3}{5}, -5$

(b) $|x - 1| = |x + 2|$
squaring both sides $\Rightarrow |x - 1|^2 = |x + 2|^2$
$(x - 1)^2 = (x + 2)^2$
$x^2 - 2x + 1 = x^2 + 4x + 4$
$6x = -3$
$x = \frac{-3}{6} = \frac{-1}{2}$

(c) $|7x + 1| = |5x + 3|$
squaring both sides $\Rightarrow |7x + 1|^2 = |5x + 3|^2$
$(7x + 1)^2 = (5x + 3)^2$
$49x^2 + 14x + 1 = 25x^2 + 30x + 9$
$\Rightarrow 24x^2 - 16x - 8 = 0$
$3x^2 - 2x - 1 = 0$
$(3x + 1)(x - 1) = 0$
$x = \frac{-1}{3}, 1$
OR
using $|x| = x$
$|7x + 1| = |5x + 3|$
$\Rightarrow 7x + 1 = 5x + 3$
$2x = 2$
$x = 1$
using $|x| = -x$
$7x + 1 = -(5x + 3)$
$7x + 1 = -5x - 3$
$12x = -4$
$x = \frac{-4}{12} = \frac{-1}{3}$

3 (a) $|x| = 2 - |x|$
$\Rightarrow 2|x| = 2$
$|x| = 1$
$\Rightarrow x = 1$ or -1

(b) $2|x| = 3 + 2x - x^2$
$|x| = x \Rightarrow 2x = 3 + 2x - x^2$
$x^2 = 3 \Rightarrow x = \sqrt{3}, -\sqrt{3}$.
$|x| = -x \Rightarrow -2x = 3 + 2x - x^2$
$x^2 - 4x - 3 = 0$

$x = \frac{4 \pm \sqrt{28}}{2} = \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}$

(c) $|x^2 - 1| - 1 = 3x - 2$

$|x| = x \Rightarrow x^2 - 1 - 1 = 3x - 2$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0, 3$

$|x| = -x \Rightarrow -(x^2 - 1) - 1 = 3x - 2$

$-x^2 + 1 - 1 = 3x - 2$

$x^2 + 3x - 2 = 0$

$x = \frac{-3 \pm \sqrt{17}}{2}$

4 $\left|\frac{2x+1}{3x-4}\right| = 2$

$\Rightarrow |2x + 1| = 2|3x - 4|$

$\Rightarrow |2x + 1|^2 = 4|3x - 4|^2$

$\Rightarrow (2x + 1)^2 = 4(3x - 4)^2$

$\Rightarrow 4x^2 + 4x + 1 = 4(9x^2 - 24x + 16)$

$\Rightarrow 4x^2 + 4x + 1 = 36x^2 - 96x + 64$

$32x^2 - 100x + 63 = 0$

$(4x - 9)(8x - 7) = 0$

$x = \frac{9}{4}, \frac{7}{8}$

5 $|4x - 1|^2 - 6|4x - 1| + 5 = 0$

Let $y = |4x - 1|$

$\Rightarrow y^2 - 6y + 5 = 0$

$(y - 1)(y - 5) = 0$

$y = 1, 5$

$|4x - 1| = 1, |4x - 1| = 5$

$4x - 1 = 1, 4x - 1 = -1, 4x - 1 = 5, 4x - 1 = -5$

$x = \frac{1}{2}, x = 0, x = \frac{3}{2}, x = -1$

6 $|3x + 2|^2 - 9|3x + 2| + 20 = 0$

$y = |3x + 2|$

$y^2 - 9y + 20 = 0$

$(y - 4)(y - 5) = 0$

$y = 4, 5$

$|3x + 2| = 4, |3x + 2| = 5$

$3x + 2 = 4, 3x + 2 = -4, 3x + 2 = 5, 3x + 2 = -5$

$x = \frac{2}{3}, x = -2, x = 1, x = \frac{-7}{3}$

7 (a) $2x^2 - 5|x| + 2 = 0$

Since $|x|^2 = x^2$

$\Rightarrow 2|x|^2 - 5|x| + 2 = 0$

$y = |x| \Rightarrow 2y^2 - 5y + 2 = 0$

$(2y - 1)(y - 2) = 0$

$y = \frac{1}{2}, 2$

$|x| = \frac{1}{2}, |x| = 2$

$\Rightarrow x = \frac{1}{2}, \frac{-1}{2}, 2, -2$

(b) $3x^2 - 19|x| + 20 = 0$

Since $|x|^2 = x^2$

$\Rightarrow 3|x|^2 - 19|x| + 20 = 0$

$y = |x|$

$\Rightarrow 3y^2 - 19y + 20 = 0$

$\Rightarrow (3y - 4)(y - 5) = 0$

$y = \frac{4}{3}, 5$

$|x| = \frac{4}{3}, \quad |x| = 5$

$\therefore x = \frac{4}{3}, \frac{-4}{3}, 5, -5$

OR:

$|x| = x$

$\Rightarrow 3x^2 - 19x + 20 = 0$

$(3x - 4)(x - 5) = 0$

$x = \frac{4}{3}, 5$

$|x| = -x$

$\Rightarrow 3x^2 + 19x + 20 = 0$

$(3x + 4)(x + 5) = 0$

$x = \frac{-4}{3}, -5$

Hence $x = \frac{4}{3}, \frac{-4}{3}, 5, -5$

8 (a) $|4x - 1| < 3$

$\Rightarrow -3 < 4x - 1 < 3$

$-2 < 4x < 4$

$\frac{-1}{2} < x < 1$

(b) $|2x + 4| < 5$

$\Rightarrow -5 < 2x + 4 < 5$

$-9 < 2x < 1$

$\frac{-9}{2} < x < \frac{1}{2}$

(c) $|3x - 1| > 6$

$\Rightarrow 3x - 1 > 6, 3x - 1 < -6$

$3x > 7, 3x < -5$

$x > \frac{7}{3} \quad x < \frac{-5}{3}$

(d) $|5x + 2| > 9$

$\Rightarrow 5x + 2 > 9, 5x + 2 < -9$

$5x > 7, 5x < -11$

$x > \frac{7}{5}$ $\quad$ $x < \frac{-11}{5}$

We can also square both sides and solve

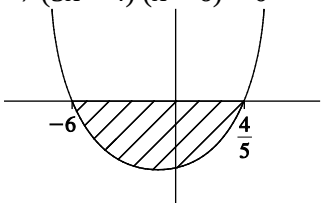
9 (a) $|3x + 1| < |2x - 5|$

$\Rightarrow (3x + 1)^2 < (2x - 5)^2$

$\Rightarrow 9x^2 + 6x + 1 < 4x^2 - 20x + 25$

$\Rightarrow 5x^2 + 26x - 24 < 0$

$\Rightarrow (5x - 4)(x + 6) < 0$



$\therefore \left\{x : -6 < x < \frac{4}{5}\right\}$

(b) $|7x + 1| < |3x + 5|$

$\Rightarrow (7x + 1)^2 < (3x + 5)^2$

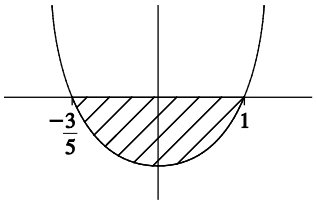
$\Rightarrow 49x^2 + 14x + 1 < 9x^2 + 30x + 25$

$\Rightarrow 40x^2 - 16x - 24 < 0$

$5x^2 - 2x - 3 < 0$

$(5x + 3)(x - 1) < 0$

$\left\{x : -\frac{3}{5} < x < 1\right\}$



(c) $4|x + 2| < |x - 1|$

$\Rightarrow 16(x + 2)^2 < (x - 1)^2$

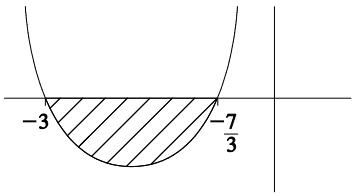
$\Rightarrow 16(x^2 + 4x + 4) < x^2 - 2x + 1$

$\Rightarrow 15x^2 + 66x + 63 < 0$

$5x^2 + 22x + 21 < 0$

$(5x + 7)(x + 3) < 0$

$\therefore \left\{x : -3 < x < -\frac{7}{3}\right\}$



Review Exercise 8

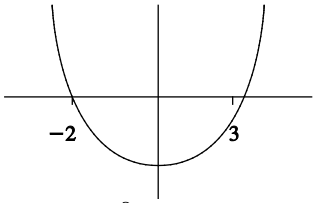
1 $2x + 1 > 4x - 5$

$2x - 4x > -5 - 1$
$-2x > -6$
$x < 3$

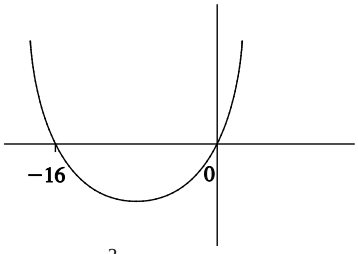
2 $x^2 - 2\sqrt{3}x > 6$
$x^2 - 2\sqrt{3}x - 6 > 0$
$x = \dfrac{2\sqrt{3} \pm \sqrt{12 + 24}}{2}$
$= \dfrac{2\sqrt{3} \pm 6}{2}$
$= \sqrt{3} \pm 3$
$(x - (\sqrt{3} + 3)\ (x - (\sqrt{3} - 3)) > 0$
$x < \sqrt{3} - 3, \quad x > \sqrt{3} + 3$

3 $(p + 3)\ x^2 - 2px + p + 2 = 0$
$b^2 - 4ac > 0$
$(-2p)^2 - 4\,(p + 3)\,(p + 2) > 0$
$4p^2 - 4p^2 - 20p - 24 > 0$
$-20p > 24$
$p < -\dfrac{24}{20} = -\dfrac{5}{4}$

4 $4x^2 - 4\lambda\, x = 5\lambda - 12x - 15$
$\Rightarrow 4x^2 - 4\lambda x + 12x - 5\lambda + 15 = 0$
$\Rightarrow 4x^2 + x\,(-4\lambda + 12) - 5\lambda + 15 = 0$
$a = 4,\ b = -4\lambda + 12,\ c = -5\lambda + 15$
$b^2 - 4ac < 0$
$(-4\lambda + 12)^2 - 4(4)\,(-5\,\lambda + 15) < 0$
$16\lambda^2 - 96\lambda + 144 + 80\lambda - 240 < 0$
$16\lambda^2 - 16\lambda - 96 < 0$
$\lambda^2 - \lambda - 6 < 0$
$(\lambda - 3)\ (\lambda + 2) < 0$
$-2 < \lambda < 3$

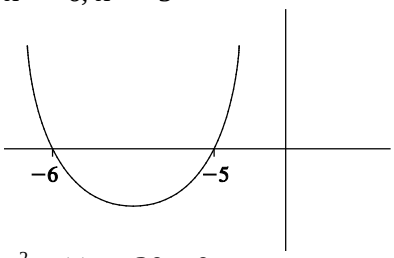


5 $(2 - 3\theta)\ x^2 = (\theta - 4)\ x - 2$
$(2 - 3\theta)\ x^2 - (\theta - 4)\ x + 2 = 0$
$b^2 - 4ac < 0$
$\Rightarrow [-(\theta - 4)]^2 - 4(2 - 3\theta)\ (2) < 0$
$\theta^2 - 8\theta + 16 - 16 + 24\theta < 0$
$\theta^2 + 16\theta < 0$
$\theta(\theta + 16) < 0$
$-16 < \theta < 0$



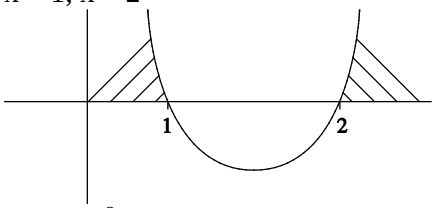
6 $(\lambda + 1)\, y^2 + (2\lambda + 3)\, y + \lambda + 2 = 0$
For real roots $b^2 - 4ac \geq 0$
$(2\lambda + 3)^2 - 4(\lambda + 1)(\lambda + 2) \geq 0$
$4\lambda^2 + 12\lambda + 9 - 4\lambda^2 - 12\lambda - 8 \geq 0$
$\Rightarrow 1 \geq 0$
$\Rightarrow \forall\ \lambda \in R\ \ b^2 - 4ac \geq 0$

7 (a) $x^2 + 11x + 30 > 0$
$(x + 5)(x + 6) > 0$
$x < -6,\ x > -5$



(b) $x^2 + 11x + 30 < 0$
$(x + 5)(x + 6) < 0$
$-6 < x < -5$

8 $4x^2 - 3x + 2 > 3x^2$
$x^2 - 3x + 2 > 0$
$(x - 1)(x - 2) > 0$
$x < 1,\ x > 2$



9 $|3x + 2|^2 - 9|3x + 2| + 20 = 0$
$y = |3x + 2|$
$y^2 - 9y + 20 = 0$
$(y - 4)(y - 5) = 0$
$y = 4, 5$
$|3x + 2| = 4, \qquad |3x + 2| = 5$
$3x + 2 = 4,\ \ 3x + 2 = -4,\ 3x + 2 = 5,\ \ 3x + 2 = -5$
$x = \frac{2}{3}, \qquad x = -2, \qquad x = 1,\ x = \frac{-7}{3}$

10 $\left|\frac{2x+1}{3x-2}\right| = 1$
$|2x + 1| = |3x - 2|$
$(2x + 1)^2 = (3x - 2)^2$
$4x^2 + 4x + 1 = 9x^2 - 12x + 4$
$5x^2 - 16x + 3 = 0$

$(5x - 1)(x - 3) = 0$

$x = \frac{1}{5}, 3$

11 $|2 - x| = 2|x + 2|$

$(2 - x)^2 = 4(x + 2)^2$

$4 - 4x + x^2 = 4[x^2 + 4x + 4]$

$4 - 4x + x^2 = 4x^2 + 16x + 16$

$3x^2 + 20x + 12 = 0$

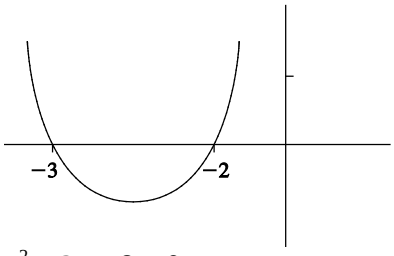
$(3x + 2)(x + 6) = 0$

$x = \frac{-2}{3}, -6$

12 (a) $x^2 + 5x + 6 < 0$

$(x + 2)(x + 3) < 0$

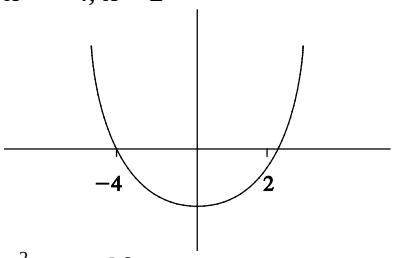
$-3 < x < -2$



(b) $x^2 + 2x - 8 > 0$

$(x + 4)(x - 2) > 0$

$x < -4, x > 2$

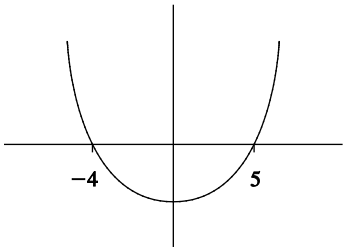


(c) $x^2 < x + 20$

$x^2 - x - 20 < 0$

$(x - 5)(x + 4) < 0$

$-4 < x < 5$



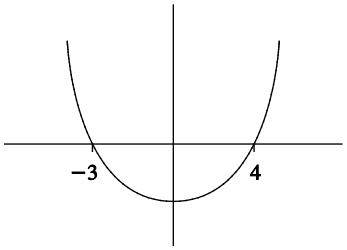
(d) $(x + 3)(x - 2) > 2(x + 3)$

$x^2 + x - 6 > 2x + 6$

$x^2 - x - 12 > 0$

$(x - 4)(x + 3) > 0$

$x < -3, x > 4$



13 $y = 3x + p$
$x^2 + y^2 = 64$
$\Rightarrow x^2 + (3x + p)^2 = 64$
$\Rightarrow x^2 + 9x^2 + 6px + p^2 = 64$
$\Rightarrow 10x^2 + 6px + p^2 - 64 = 0$
For no real values of x, $b^2 - 4ac < 0$
$(6p)^2 - 4(10)(p^2 - 64) < 0$
$36p^2 - 40p^2 + 2560 < 0$
$4p^2 - 2560 > 0$
$p^2 - 640 > 0$
$(p - \sqrt{640})(p + \sqrt{640}) > 0$
$\{p : p < -\sqrt{640}\} \cup \{p : p > \sqrt{640}\}$

14 $y = 3 + px$
$x^2 + 2xy + 1 = 0$
$\Rightarrow x^2 + 2x(3 + px) + 1 = 0$
$x^2 + 6x + 2px^2 + 1 = 0$
$x^2(1 + 2p) + 6x + 1 = 0$
For no real roots $b^2 - 4ac < 0$
$(6)^2 - 4(1 + 2p)(1) < 0$
$36 - 4 - 8p < 0$
$32 < 8p$
$4 < p$

15 $\frac{2x+1}{3x-4} < 2$

$\frac{2x+1}{3x-4} - 2 < 0$

$\frac{2x+1-6x+8}{3x-4} < 0$

$\frac{-4x+9}{3x-4} < 0$

Critical values are $\frac{4}{3}, \frac{9}{4}$

	$-4x + 9$	$3x - 4$	$\frac{-4x+9}{3x-4}$
$x < \frac{4}{3}$	+ve	–ve	+ve
$\frac{4}{3} < x < \frac{9}{4}$	+ve	+ve	+ve
$x > \frac{9}{4}$	–ve	+ve	–ve

$x < \frac{4}{3}, x > \frac{9}{4}$

16 $f(x) > 1$

$\Rightarrow \frac{2-x}{x+3} > 1$

$\Rightarrow \frac{2-x}{x+3} - 1 > 0$

$\Rightarrow \frac{2-x-x-3}{x+3} > 0$

$\Rightarrow \frac{-2x-1}{x+3} > 0$

$\frac{2x+1}{x+3} < 0$

The critical values are $\frac{-1}{2}, -3$

	2x+1	x+3	$\frac{2x+1}{x+3}$
$x < -3$	–ve	–ve	+ve
$-3 < x < \frac{-1}{2}$	–ve	+ve	–ve
$x > \frac{-1}{2}$	+ve	+ve	+ve

$\therefore -3 < x < -\frac{1}{2}$

17 (a) $2x^2 - 5|x| + 2 = 0$

$2|x|^2 - 5|x| + 2 = 0$

$(2|x| - 1)(|x| - 2) = 0$

$|x| = \frac{1}{2}, \quad |x| = 2$

$x = \frac{1}{2}, \frac{-1}{2}, x = 2, -2$

(b) $3x^2 - 19|x| + 20 = 0$

$3|x|^2 - 19|x| + 20 = 0$

$(3|x| - 4)(|x| - 5) = 0$

$|x| = \frac{4}{3}, \quad |x| = 5$

$x = \frac{4}{3}, \frac{-4}{3}, \quad 5, -5$

18 $\frac{4x+2}{x-1} + 2 > 0$

$\frac{4x+2+2x-2}{x-1} > 0$

$\frac{6x}{x-1} > 0$

The critical values are 0, 1

	$6x$	$x-1$	$\frac{6x}{x-1}$
$x<0$	–ve	–ve	+ve
$0<x<1$	+ve	–ve	–ve
$x>1$	+ve	+ve	+ve

$\{x : x < 0\} \cup \{x : x > 1\}$

19 $y = \frac{p - x}{2}$

$y = \frac{-8}{x} - x$

$\frac{p}{2} - \frac{x}{2} = \frac{-8}{x} - x$

$\times 2x \Rightarrow px - x^2 = -16 - 2x^2$

$x^2 + px + 16 = 0$

$b^2 - 4ac \geq 0$

$p^2 - 4(1)(16) \geq 0$

$p^2 - 64 \geq 0$

$(p - 8)(p + 8) \geq 0$

$p \leq -8, \quad p \geq 8$

20 $s = -16t^2 + 80t + 6$

$s > 6 \Rightarrow -16t^2 + 80t + 6 > 6$

$\Rightarrow -16t^2 + 80t > 0$

$\Rightarrow -16t(t - 5) > 0$

$\Rightarrow 0 < t < 5$

The ball will be above 6 metres for anywhere between 0 and 5 seconds.

21 $P = 8x - 0.02x^2$

$P > 4$

$\Rightarrow 8x - 0.02x^2 > 4$

$\Rightarrow 0.02x^2 - 8x + 4 < 0$

$\Rightarrow x^2 - 400x + 200 < 0$

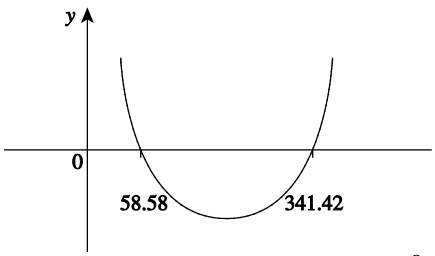
When $x^2 - 400x + 200 = 0$

$x = \frac{400 \pm \sqrt{(-400)^2 - 4(200)}}{2}$

$= 399.5, 0.5$

$\Rightarrow (x - 341.42)(x - 58.58) < 0$

$\therefore 58.58 < x < 341.42$



When $x = 58$, $p = 8(58) - 0.02(58)^2 = 39.72$

$x = 59$, $p = 8(59) - 0.02(59)^2 = 402.38$

$x = 341$, $p = 8(341) - 0.02(341)^2 = 402.38$

$x = 342$, $p = 8(342) - 0.02(342)^2 = 396.72$

$\therefore 59 \leq \underline{x} \, 341$

22 $y = -4x^2 + 4\,000x$

(a) $y = 0 \Rightarrow -4x^2 + 4\,000x = 0$
$-4x(x - 1\,000) = 0$
$x = 0, 1\,000$
$\therefore$ Revenue is \$0 when x = \$0 and x = \$1 000.00

(b) When $y > 800\,000$
$-4x^2 + 4\,000x > 800\,000$
$\Rightarrow -4x^2 + 4\,000x - 800\,000 > 0$
$\Rightarrow x^2 - 1\,000\,x + 200\,000 < 0$
Now $x^2 - 1\,000\,x + 200\,000 = 0$

$$\Rightarrow x = \frac{1\,000 \pm \sqrt{(-1000)^2 - 4\,(200\,000)}}{2}$$

$$x = \frac{1000 \pm 447.21}{2}$$

$= 723.61, 276.40$
$\therefore (x - 723.61)(x - 276.40) < 0$
$\therefore 276.40 < x < 723.61$
The range of prices is
\$276.40 < x < \$723.61

