Chapter 4 Polynomials

Try these 4.1

(a) $(x - 2)(x + 1)(x + 3)$
$= (x^2 - x - 2)(x + 3)$
$= x^3 + 3x^2 - x^2 - 3x - 2x - 6$
$= x^3 + 2x^2 - 5x - 6$
$\Rightarrow x^3 + 2x^2 - 5x - 6 = Ax^3 + Bx^2 + Cx + D$
$\Rightarrow A = 1, B = 2, C = -5, D = -6$

(b) $2x^2 - 3x + 1 = A(x + 1)^2 + Bx + C$
$= A(x^2 + 2x + 1) + Bx + C$
$= Ax^2 + 2Ax + Bx + A + C$
Equating coefficients
$\Rightarrow A = 2$
$2A + B = -3$
$4 + B = -3$
$B = -7$
$A + C = 1$
$2 + C = 1$
$C = -1$
$\therefore A = 2, B = -7, C = -1$

Exercise 4A

1 $x^2 + x + b \equiv (x + b)(x - 2) + a$
$x^2 + x + b = x^2 - 2x + bx - 2b + a$
Equating coefficients of x
$\Rightarrow 1 = -2 + b \Rightarrow b = 3$
Equating constants:
$b = -2b + a$
$3 = -6 + a$
$a = 9$
$\therefore a = 9, b = 3$

2 $4x^2 + 6x + 1 = p(x + q)^2 + r$
$4x^2 + 6x + 1 = p[x^2 + 2qx + q^2] + r$
Coeff of $x^2 \Rightarrow 4 = p$
Coeff of $x \Rightarrow 6 = 2pq, 6 = 2(4)q$
$q = \frac{6}{8} = \frac{3}{4}$
Constants $\Rightarrow 1 = pq^2 + r$
$\Rightarrow 1 = 4\left(\frac{3}{4}\right)^2 + r$
$r = 1 - \frac{9}{4} = -\frac{5}{4}$
Hence $p = 4,\ q = \frac{3}{4},\ r = \frac{-5}{4}$

3 $8x^3 + 27x^2 + 49x + 15 = (ax + 3)(x^2 + bx + c)$
$= ax^3 + abx^2 + acx + 3x^2 + 3bx + 3c$ $\quad (8x + 3)(x^2 + 3x + 5)$

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$= ax^3 + x^2 (ab + 3) + x (ac + 3b) + 3c$

$8x^3 + 24x^2 + 40x + 3x^2 + 9x + 15$
$= 8x^2 + 27x^2 + 49x + 15$

Equating coeff of x^3: $a = 8$
Equating coeff of x^2: $ab + 3 = 27 \Rightarrow 8b + 3 = 27 \Rightarrow b = 3$
Coeff of x: $ac + 3b = 49 \Rightarrow 8c + 9 = 49 \Rightarrow c = 5$
Hence $a = 8, b = 3, c = 5$

4 $x^3 + px^2 - 7x + 6 = (x - 1)(x - 2)(qx + r)$
Coeff of $x^3 \Rightarrow q = 1$
Constants $\Rightarrow 6 = (-1)(-2)r \Rightarrow r = 3$
Coeff of $x^2 \Rightarrow p = -3q + r \Rightarrow p = -3 + 3 = 0$
Hence $p = 0, q = 1, r = 2$

5 $2x^3 + 7x^2 - 7x - 30 = (x - 2)(ax^2 + bx + c)$
$= ax^3 + bx^2 + cx - 2ax^2 - 2bx - 2c$
$= ax^3 + x^2 (b - 2a) + x (c - 2b) - 2c$
Equating coeff of $x^3 \Rightarrow a = 2$
Equating coeff of $x^2 \Rightarrow b - 2a = 7 \Rightarrow b = 11$
Equating coeff of $x \Rightarrow c - 2b = -7 \Rightarrow c = 15$
Hence $a = 2, b = 11, c = 15$

6 $x^3 - 3x^2 + 4x + 2 \equiv (x - 1)(x^2 - 2x + a) + b$
$= x^3 - 2x^2 + ax - x^2 + 2x - a + b$
$= x^3 - 3x^2 + x(a + 2) + b - a$
Equating coeff of $x \Rightarrow 4 = a + 2$
$a = 2$
Equating constants $\Rightarrow 2 = b - a$
$b = 4$
Hence $a = 2$, $b = 4$

7 $4x^3 + 3x^2 + 5x + 2 = (x + 2)(ax^2 + bx + c)$
Equating coeff of x^3: $a = 4$
Equating constants: $2 = 2c \Rightarrow c = 1$
Coeff of x^2: $3 = 2a + b \Rightarrow 3 = 8 + b$
$b = -5$
Hence $a = 4, b = -5, c = 1$

8 $2x^3 + Ax^2 - 8x - 20 = (x^2 - 4)(Bx + C)$
$= Bx^3 + Cx^2 - 4Bx - 4C$
Coeff of $x^3 \Rightarrow B = 2$
Coeff of $x^2 \Rightarrow C = A$
Coeff of $x \Rightarrow -8 = -4B \Rightarrow B = 2$
Constants $\Rightarrow -20 = -4C, C = 5$
Hence $A = 5, B = 2, C = 5$

9 $ax^3 + bx^2 + cx + d = (x + 2)(x + 3)(x + 4)$
$= (x^2 + 5x + 6)(x + 4)$
$= x^3 + 4x^2 + 5x^2 + 20x + 6x + 24$
$= x^3 + 9x^2 + 26x + 24$
$\therefore a = 1, b = 9, c = 26, d = 24$

10 $ax^3 + bx^2 + cx + d = (4x + 1)(2x - 1)(3x + 2)$
$= (8x^2 - 2x - 1)(3x + 2)$
$= 24x^3 + 16x^2 - 6x^2 - 4x - 3x - 2$
$= 24x^3 + 10x^2 - 7x - 2$
$\therefore a = 24, b = 10, c = -7, d = -2$

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$$\begin{array}{r} 4x^2+5x+12 \\ x-2\,\overline{\big)\,4x^3-3x^2+2x+1} \\ \underline{-(4x^3-8x^2)} \\ 5x^2+2x+1 \\ \underline{5x^2-10x} \\ 12x+1 \\ \underline{12x-24} \\ 25 \end{array}$$

∴ Quotient is $4x^2 + 5x + 12$
Remainder is 25.

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$$\begin{array}{r} 5x^2+4x+10 \\ x-2\,\overline{\big)\,5x^3-6x^2+2x+1} \\ \underline{-(5x^3-10x^2)} \\ 4x^2+2x+1 \\ \underline{4x^2-8x} \\ 10x+1 \\ \underline{10x-20} \\ 21 \end{array}$$

$$\therefore \frac{5x^3-6x^2+2x+1}{x-2}=5x^2+4x+10+\frac{21}{x-2}$$

∴ A = 5, B = 4, C = 10

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$$\begin{array}{r} x^3-2x^2-2x+3 \\ x^2+1\,\overline{\big)\,x^5-2x^4-x^3+x^2+x+1} \\ \underline{-(x^5+x^3)} \\ -2x^4-2x^3+x^2+x+1 \\ \underline{-2x^4-2x^2} \\ -2x^3+3x^2+x+1 \\ \underline{-2x^3-2x} \\ 3x^2+3x+1 \\ \underline{3x^2+3} \\ 3x-2 \end{array}$$

∴ Quotient is $x^3 – 2x^2 – 2x + 3$
Remainder is $3x – 2$

14 (a) $\frac{2}{x+1}+\frac{3}{x-2}$

$$=\frac{2(x-2)+3(x+1)}{(x+1)(x-2)}$$

$$=\frac{2x-4+3x+3}{(x+1)(x-2)}$$

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$$= \frac{5x - 1}{(x+1)(x-2)}$$

(b) $$\frac{x+1}{x+3} - \frac{2x+1}{2x-4}$$

$$= \frac{(x+1)(2x-4) - (2x+1)(x+3)}{(x+3)(2x-4)}$$

$$= \frac{\cancel{2x^2} - 2x - 4 - \cancel{2x^2} - 7x - 3}{(x+3)(2x-4)}$$

$$= \frac{-9x - 7}{(x+3)(2x-4)}$$

(c) $$\frac{x}{x^2 + 2x + 1} - \frac{x-1}{x+2}$$

$$= \frac{x(x+2) - (x-1)(x^2 + 2x + 1)}{(x^2 + 2x + 1)(x+2)}$$

$$= \frac{\cancel{x^2} + 2x - x^3 - \cancel{2x^2} - x + \cancel{x^2} + 2x + 1}{(x^2 + 2x + 1)(x+2)}$$

$$= \frac{-x^3 + 3x + 1}{(x^2 + 2x + 1)(x+2)}$$

(d) $$\frac{3x+4}{x-1} - \frac{x}{x+1} + \frac{x+2}{2x+1}$$

$$= \frac{(3x+4)(x+1)(2x+1) - x(x-1)(2x+1) + (x+2)(x-1)(x+1)}{(x-1)(x+1)(2x+1)}$$

$$= \frac{6x^3 + 17x^2 + 15x + 4 - 2x^3 + x^2 + x + x^3 + 2x^2 - x - 2}{(x-1)(x+1)(2x+1)}$$

$$= \frac{5x^3 + 20x^2 + 15x + 2}{(x+1)(x-1)(2x+1)}$$

(e) $$\frac{x^2}{2-x} - \frac{x^2}{3-x}$$

$$= \frac{x^2(3-x) - x^2(2-x)}{(2-x)(3-x)}$$

$$= \frac{3x^2 \cancel{-x^3} - 2x^2 + \cancel{x^3}}{(2-x)(3-x)} = \frac{x^2}{(2-x)(3-x)}$$

Try these 4.2

(a) Let $f(x) = 6x^3 - 3x^2 + x - 2$

(i) $f(2) = 6(2)^3 - 3(2)^2 + 2 - 2$
$= 48 - 12 + 2 - 2$
$= 36$

(ii) $f(-1) = 6(-1)^3 - 3(-1)^2 + (-1) - 2$
$= -6 - 3 - 1 - 2$
$= -12$

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(iii) $f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + \frac{1}{2} - 2$

$= \frac{3}{4} - \frac{3}{4} + \frac{1}{2} - 2 = -\frac{3}{2}$

(b) $f(x) = x^4 + ax^2 - 2x + 1$

$f(1) = 4$

$\Rightarrow 1 + a - 2 + 1 = 4$

$a = 4$

(c) $f(x) = x^3 - 4x^2 + ax + b$

$f\left(\frac{1}{2}\right) = 1 \Rightarrow \frac{1}{8} - 1 + \frac{1}{2}a + b = 1$

$\therefore \frac{1}{2}a + b = \frac{15}{8}$ …(1)

$f(1) = 2 \Rightarrow 1 - 4 + a + b = 2$

$a + b = 5$ …(2)

$(2) - (1) \Rightarrow \frac{1}{2}a = \frac{25}{8}$

$a = \frac{25}{4} = 6\frac{1}{4}$

$6\frac{1}{4} + b = 5$

$b = \frac{-5}{4}$

$\therefore a = 6\frac{1}{4},\ b = -1\frac{1}{4}$

Exercise 4B

1 (a) $f(x) = ax^4 + 3x^2 - 2x + 1$

$f(1) = a + 3 - 2 + 1$

$= a + 2.$

(b) $f(x) = 3x^3 + 6x^2 - 7x + 2$

$f(-1) = 3(-1)^3 + 6(-1)^2 - 7(-1) + 2$

$= 12$

(c) $f(x) = x^5 + 6x^2 - x + 1$

$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^5 + 6\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 1$

$= \frac{95}{32}$

(d) $f(x) = (4x + 2)(3x^2 + x + 2) + 7$

$f(2) = (10)(12 + 2 + 2) + 7$

$= 167$

(e) $f(x) = x^7 + 6x^2 + 2$

$f(-2) = (-2)^7 + 6(-2)^2 + 2$

$= -102$

(f) $f(x) = 4x^3 - 3x^2 + 5$

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$f\left(-\frac{3}{2}\right) = 4\left(\frac{-3}{2}\right)^3 - 3\left(\frac{-3}{2}\right)^2 + 5$

$= \frac{-61}{4}$

(g) $f(x) = 3x^4 - 4x^3 + x^2 + 1$
$f(3) = 3(3)^4 - 4(3)^3 + (3)^2 + 1$
$= 145$

2 $f(2) = a$
$\Rightarrow 2^2 - a(2) + 2 = a$
$6 = 3a$
$a = 2$

3 $f(x) = 5x^2 - 4x + b$
$f\left(-\frac{1}{2}\right) = 2$

$\Rightarrow 5\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + b = 2$

$b = \frac{-5}{4}$

4 $f(x) = 3x^3 + ax^2 + bx + 1$
$f(1) = 2$
$\Rightarrow 3 + a + b + 1 = 2$
$a + b = -2$ [1]
$f(2) = 13$
$\Rightarrow 3(2)^3 + a(2)^2 + b(2) + 1 = 13$
$\Rightarrow 4a + 2b = -12$
$2a + b = -6$ [2]
[2] – [1] $\Rightarrow a = -4$
$b = 2$
$\therefore a = -4, b = 2$

5 $f(x) = x^3 + px^2 + qx + 2$
$f(-1) = -3$
$\Rightarrow -1 + p - q + 2 = -3$
$p - q = -4$ [1]
$f(2) = 54$
$(2)^3 + p(2)^2 + q(2) + 2 = 54$
$4p + 2q = 44$
$2p + q = 22$ [2]
[1] + [2] $\Rightarrow 3p = 18, p = 6$
$q = 10$
$\therefore f(x) = x^3 + 6x^2 + 10x + 2$

$f\left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)^3 + 6\left(\frac{-1}{2}\right)^2 + 10\left(\frac{-1}{2}\right) + 2 = \frac{-13}{8}$

6 $f(x) = 2x^3 - 3x^2 - 4x + 1$
$f(a) = f(-a)$
$2a^3 - 3a^2 - 4a + 1 = -2a^3 - 3a^2 + 4a + 1$
$\Rightarrow 4a^3 - 8a = 0$
$\Rightarrow 4a(a^2 - 2) = 0$
$a = 0, a^2 = 2 \Rightarrow a = \sqrt{2}, -\sqrt{2}$

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Hence a = 0, $\sqrt{2}$, $-\sqrt{2}$

7 $f(x) = 2x^3 - x^2 - 2x - 1$

$f(2) = 2\,f(2a)$

$\Rightarrow 2(2)^3 - (2)^2 - 2\,(2) - 1 = 2\,[2(2a)^3 - (2a)^2 - 2\,(2a) - 1]$

$\Rightarrow 16 - 4 - 4 - 1 = 32\,a^3 - 8a^2 - 8a - 2$

$32\,a^3 - 8a^2 - 8a - 9 = 0$

8 $f\,(x) = 2x^3 - 5x^2 - 4x + b$

$f\,(-2) = 2\,f(1)$

$2\,(-2)^3 - 5\,(-2)^2 - 4\,(-2) + b = 2\,[2 - 5 - 4 + b]$

$\Rightarrow -16 - 20 + 8 + b = -14 + 2b$

$b = -14$

9 $f\,(x) = x^3 + (\lambda + 5)\,x + \lambda$

$f(1) + f\,(-2) = 0$

$f\,(1) = 1 + \lambda + 5 + \lambda = 2\,\lambda + 6$

$f\,(-2) = -8 - 2\,(\lambda + 5) + \lambda = -\lambda - 18$

$2\lambda + 6 - \lambda - 18 = 0$

$\lambda - 12 = 0$

$\lambda = 12$

10 $f(x) = 3x^3 + kx^2 + 15$

$f(3) = 3\,(3)^3 + k(3)^2 + 15$

$= 9k + 96$

$$f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 + k\left(\frac{1}{3}\right)^2 + 15$$

$$= \frac{1}{9}k + \frac{136}{9}$$

$$f(3) = \frac{1}{3}f\left(\frac{1}{3}\right)$$

$$9K + 96 = \frac{1}{27}k + \frac{136}{27}$$

$$9K - \frac{1}{27}k = \frac{136}{27} - 96$$

$$\frac{242}{27}k = \frac{-2456}{27}$$

$$k = \frac{-1228}{121}$$

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$$\begin{array}{r|l} & 3x + (p-6) \\ \hline x^2 + 2x + 3 & 3x^3 + px^2 + qx + 2 \\ - & \underline{3x^3 + 6x^2 + 9x} \\ & (p-6)\,x^2 + (q-9)\,x + 2 \\ & \underline{(p-6)\,x^2 + 2\,(p-6)\,x + 3\,(p-6)} \\ & [(q-9) - 2\,(p-6)]\,x + 2 - 3(p-6) \end{array}$$

$\therefore\ (q - 9 - 2p + 12)\,x + (20 - 3p) = x + 5$

$\therefore\ 20 - 3p = 5 \Rightarrow p = 5$

$q - 2p + 3 = 1$

$q - 10 + 3 = 1$

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q = 8
p= 5, q = 8

12 f (x) = $8x^3 + px^2 + qx + 2$

$f\left(\frac{1}{2}\right) = \frac{7}{2}$

$\Rightarrow 8\left(\frac{1}{2}\right)^3 + P\left(\frac{1}{2}\right)^2 + q\left(\frac{1}{2}\right) + 2 = \frac{7}{2}$

$\Rightarrow \frac{1}{4}p + \frac{1}{2}q = \frac{1}{2}$

f (–1) = –1

$\Rightarrow 8(-1)^3 + p(-1)^2 + q(-1) + 2 = -1$

p – q = 5 [1]

$\frac{1}{2}$ p + q = 1 [2]

[1] +[2] $\Rightarrow \frac{3}{2}$ p= 6

p = 4
q = –1
∴ p = 4, q = – 1

13 f (x) = $6x^5 + 4x^3 - ax + 2$

f(– 1) = 15

$6(-1)^5 + 4(-1)^3 - a(-1) + 2 = 15$

⇒– 6 – 4 + a + 2 = 15

a = 23

∴ f (x) = $6x^5 + 4x^3 - 23x + 2$

f (2) = $6(2)^5 + 4(2)^3 - 23(2) + 2$

= 180

Remainder is 180

Try these 4.3

(a) Let f (x) = $3x^3 - x^2 - 3x + 1$

(i) f (1) = 3 – 1 – 3 + 1 = 0

Since f (1) = 0 ⇒ x – 1 is a factor of f (x)

(ii) $f\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) + 1$

$= \frac{-3}{8} - \frac{1}{4} + \frac{3}{2} + 1$

$= \frac{15}{8}$

Since $f\left(-\frac{1}{2}\right) \neq 0 \Rightarrow 2x + 1$ is not a factor of f (x)

(iii) $f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) + 1$

$= \frac{1}{9} - \frac{1}{9} - 1 + 1$

= 0

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Since $f\left(\frac{1}{3}\right) = 0 \Rightarrow 3x - 1$ is a factor of f (x)

(b) Let $f(x) = 4x^3 + px^2 - qx - 6$

$f\left(-\frac{1}{4}\right) = 4\left(-\frac{1}{4}\right)^3 + p\left(-\frac{1}{4}\right)^2 - q\left(-\frac{1}{4}\right) - 6$

$= -\frac{1}{16} + \frac{1}{16}p + \frac{1}{4}q - 6$

$\therefore \frac{1}{16}p + \frac{1}{4}q - 6\frac{1}{16} = 0$

$\frac{1}{16}p + \frac{1}{4}q = \frac{97}{16}$

$p + 4q = 97$ [1]

$f(1) = -20$

$\Rightarrow 4 - p + q - 6 = -20$

$p - q = -8$ [2]

$[1] - [2] \Rightarrow 5q = 115$

$q = \frac{115}{5} = 23$

$p + 92 = 97$

$p = 5$

Hence p = 5, q = 23

Try these 4.4

(a) Let $f(x) = 2x^3 + 5x^2 - 4x - 3$

$f(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3$

$= 2 + 5 - 4 - 3 = 0$

Since $f(1) = 0 \Rightarrow x - 1$ is a factor of f(x)

$$
\begin{array}{r|l}
 & 2x^2 + 7x + 3 \\
\hline
x - 1 & 2x^3 + 5x^2 - 4x - 3 \\
 & \underline{2x^3 - 2x^2} \\
 & 7x^2 - 4x - 3 \\
 & \underline{7x^2 - 7x} \\
 & 3x - 3 \\
 & \underline{3x - 3} \\
 & 0
\end{array}
$$

$\therefore 2x^3 + 5x^2 - 4x - 3 = (x - 1)(2x^2 + 7x + 3)$

$= (x - 1)(2x + 1)(x + 3)$

$\Rightarrow (x - 1)(2x + 1)(x + 3) = 0$

$\therefore x - 1 = 0, 2x + 1 = 0, x + 3 = 0$

Hence $x = 1, -\frac{1}{2}, -3$

(b) $f(x) = x^3 - 2x^2 - 5x + 6$

$f(1) = 1 - 2 - 5 + 6 = 0$

$\therefore x - 1$ is a factor of f (x)

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$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x-1 & x^3 - 2x^2 - 5x + 6 \\ & \underline{x^3 - x^2} \\ & \quad -x^2 - 5x + 6 \\ & \quad \underline{-x^2 + x} \\ & \qquad -6x + 6 \\ & \qquad \underline{-6x + 6} \\ & \qquad\quad 0 \end{array}$$

$\therefore$ f(x) = (x – 1) (x^2 – x – 6) = (x – 1) (x – 3) (x + 2)

f(x) = 0 $\Rightarrow$ x – 1 = 0, x – 3 = 0, x + 2 = 0

x = 1, 3, – 2

$\therefore$ factors of f (x) are (x – 1), (x – 3), (x + 2)

Roots of f (x) = 0 are 1, 3, – 2

Exercise 4c

1 f(x) = $x^3 + 2x^2 - x - 2$.

(a) By trial and error:

f(1) = 1 + 2 – 1 – 2 = 0

$\therefore$ x – 1 is a factor of f (x)

$$\begin{array}{r|l} & x^2 + 3x + 2 \\ \hline x - 1 & x^3 + 2x^2 - x - 2 \\ & \underline{x^3 - x^2} \\ & \quad 3x^2 - x - 2 \\ & \quad \underline{3x^2 - 3x} \\ & \qquad 2x - 2 \\ & \qquad \underline{2x - 2} \\ & \qquad\quad 0 \end{array}$$

$\therefore$ f(x) = (x – 1) (x^2 + 3x + 2)

= (x – 1) (x + 1) (x + 2)

(b) f(x) = $x^3 + 6x^2 + 11x + 6$

f(– 2) = $(-2)^3 + 6(-2)^2 + 11(-2) + 6$

= –8 + 24 – 22 + 6

= 0

$\therefore$ x + 2 is a factor of f (x)

$$\begin{array}{r|l} & x^2 + 4x + 3 \\ \hline x + 2 & x^3 + 6x^2 + 11x + 6 \\ & \underline{x^3 + 2x^2} \\ & \quad 4x^2 + 11x + 6 \\ & \quad \underline{4x^2 + 8x} \\ & \qquad 3x + 6 \\ & \qquad \underline{3x + 6} \\ & \qquad\quad 0 \end{array}$$

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$\therefore$ f (x) = (x + 2) (x^2 + 4x + 3)
= (x + 2) (x + 1) (x + 3)

(c) f (x) = x^3 – 7x + 6
f (1) = 1 – 7 + 6 = 0
$\therefore$ x – 1 is factor of f (x)

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x - 1 & x^3 - 7x + 6 \\ & \underline{x^3 - x^2} \\ & \quad x^2 - 7x + 6 \\ & \quad \underline{x^2 - x} \\ & \qquad -6x + 6 \\ & \qquad \underline{-6x + 6} \\ & \qquad\quad 0 \end{array}$$

$\therefore$ f (x) = (x – 1) (x^2 + x – 6)
= (x – 1) (x + 3) (x – 2)

(d) f (x) = x^3 – 4 x^2 + x + 6
f (2) = 2^3 – 4 $(2)^2$ + 2 + 6
= 8 – 16 + 2 + 6
= 0
⇒ x – 2 is a factor of f(x)

$$\begin{array}{r|l} & x^2 - 2x - 3 \\ \hline x - 2 & x^3 - 4x^2 + x + 6 \\ & \underline{x^3 - 2x^2} \\ & \quad -2x^2 + x + 6 \\ & \quad \underline{-2x^2 + 4x} \\ & \qquad -3x + 6 \\ & \qquad \underline{-3x + 6} \\ & \qquad\quad 0 \end{array}$$

f (x) = (x – 2) (x^2 – 2x – 3)
= (x – 2) (x – 3) (x + 1)

(e) f (x) = x^3 – 7x – 6
f (–1) = – 1 + 7 – 6 = 0
x + 1 is a factor of f (x)

$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x + 1 & x^3 - 7x - 6 \\ & \underline{x^3 + x^2} \\ & \quad -x^2 - 7x - 6 \\ & \quad \underline{-x^2 - x} \\ & \qquad -6x - 6 \\ & \qquad \underline{-6x - 6} \\ & \qquad\quad 0 \end{array}$$

$\therefore$ f (x) = (x + 1) (x^2 – x – 6)
= (x + 1) (x – 3) (x + 2)

(f) f(x) = $6x^3$ + 31 x^2 + 3x –10

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$f(-5) = 6(-5)^3 + 31(-5)^2 + 3(-5) - 10$
$= -750 + 775 - 15 - 10$
$= 0$

$$\begin{array}{r|l} & 6x^2 + x - 2 \\ \hline x + 5 & 6x^3 + 31x^2 + 3x - 10 \\ & \underline{6x^3 + 30x^2} \\ & \quad x^2 + 3x - 10 \\ & \quad \underline{x^2 + 5x} \\ & \qquad -2x - 10 \\ & \qquad \underline{-2x - 10} \\ & \qquad\quad 0 \end{array}$$

$\therefore f(x) = (x + 5)(6x^2 + x - 2)$
$= (x + 5)(3x + 2)(2x - 1)$

2 (a) $3x^3 + x^2 - 20x + 12 = 0$
$f(x) = 3x^3 + x^2 - 20x + 12$
$f(2) = 3(2)^3 + (2)^2 - 20(2) + 12$
$= 24 + 4 - 40 + 12 = 0$
$\therefore x - 2$ is a factor of $f(x)$
Now $3x^3 + x^2 - 20x + 12 = (x - 2)(ax^2 + bx + c)$
Equating coeff:
$x^3 \Rightarrow 3 = a$
$x^2 \Rightarrow 1 = -2a + b \Rightarrow b = 7$
Constants $\Rightarrow 12 = -2c \Rightarrow c = -6$
$\therefore 3x^3 + x^2 - 20x + 12 = (x - 2)(3x^2 + 7x - 6)$
$= (x - 2)(3x - 2)(x + 3)$
$\therefore (x - 2)(3x - 2)(x + 3) = 0$
$\Rightarrow x - 2 = 0, 3x - 2 = 0, x + 3 = 0$
Hence $x = 2, x = \dfrac{2}{3}, x = -3$

(b) $2x^3 + 13x^2 + 17x - 12 = 0$
$f(x) = 2x^3 + 13x^2 + 17x - 12$
$f(-4) = 2(-4)^3 + 13(-4)^2 + 17(-4) - 12$
$= -128 + 208 - 68 - 12$
$= 0$
$\therefore x + 4$ is a factor of $f(x)$
$2x^3 + 13x^2 + 17x - 12 = (x + 4)(ax^2 + bx + c)$
$a = 2, 4c = -12 \Rightarrow c = -3$
$13 = 4a + b \Rightarrow b = 5$
$\therefore f(x) = (x + 4)(2x^2 + 5x - 3)$
$= (x + 4)(2x - 1)(x + 3)$
$(x + 4)(2x - 1)(x + 3) = 0$
$\Rightarrow x + 4 = 0, 2x - 1 = 0, x + 3 = 0$
$x = -4, \dfrac{1}{2}, -3$

(c) $2x^3 - 11x^2 + 3x + 36 = 0$
$f(x) = 2x^3 - 11x^2 + 3x + 36$
$f(4) = 2(4)^3 - 11(4)^2 + 3(4) + 36$
$= 128 - 176 + 12 + 36$
$= 0$

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$x - 4$ is a factor of f (x)
Now $2x^3 - 11x^2 + 3x + 36 = (x - 4)(ax^2 + bx + c)$
x^3: a = 2
Constants: $36 = -4c \Rightarrow c = -9$
x^2: $-11 = -4a + b \Rightarrow b = -3$
$2x^3 - 11x^2 + 3x + 36 = (x - 4)(2x^2 - 3x - 9)$
$= (x - 4)(2x + 3)(x - 3)$
$(x - 4)(2x + 3)(x - 3) = 0$
$\Rightarrow x - 4 = 0, 2x + 3 = 0, x - 3 = 0$
$x = 4,\ x = \frac{-3}{2}, x = 3$

(d) $f(x) = 3x^3 + 10x^2 + 9x + 2$
$f(-1) = 3(-1)^3 + 10(-1)^2 + 9(-1) + 2$
$= -3 + 10 - 9 + 2$
$= 0.$
$x + 1$ is a factor of f (x)
$3x^3 + 10x^2 + 9x + 2 = (x + 1)(ax^2 + bx + c)$
a = 3, c = 2
$10 = a + b \Rightarrow b = 7$
$\therefore\ 3x^3 + 10x^2 + 9x + 2 = (x + 1)(3x^2 + 7x + 2)$
$= (x + 1)(3x + 1)(x + 2)$
$(x + 1)(3x + 1)(x + 2) = 0$
$x + 1 = 0, 3x + 1 = 0, x + 2 = 0$
$x = -1,\ x = -\frac{1}{3}, x = -2$

3 (a) $f(x) = x^3 + 2x^2 + 2x + 1$
$f(-1) = (-1)^3 + 2(-1)^2 + 2(-1) + 1$
$= -1 + 2 - 2 + 1 = 0$
True

(b) $f(x) = 2x^4 + 3x^2 - x + 2$
$f(1) = 2 + 3 - 1 + 2 = 6$
Since $f(1) \neq 0$
False

(c) $f(x) = x^5 - 4x^4 + 3x^3 - 2x^2 + 4$
$f(2) = (2)^5 - 4(2)^4 + 3(2)^3 - 2(2)^2 + 4$
$= 32 - 64 + 24 - 8 + 4$
$= -12$
False

(d) $f(x) = 2x^3 + 9x^2 + 10x + 3$
$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + 9\left(-\frac{1}{2}\right)^2 + 10\left(-\frac{1}{2}\right) + 3 = 0$$
True

4 $f(x) = x^3 - 3x^2 + kx + 2$
$f(1) = 1 - 3 + k + 2 = 0$
$\Rightarrow k = 0.$

5 $f(x) = x^3 - 12x + 16$
$f(2) = (2)^3 - 12(2) + 16$
$= 0.$
$\therefore\ x - 2$ is a factor of f(x)
$x^3 - 12x + 16 = (x - 2)(ax^2 + bx + c)$
$a = 1, -2c = 16$

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c = – 8
0 = – 2a + b, b = 2a = 2
$\therefore$ $x^3 - 12x + 16 = (x - 2)(x^2 + 2x - 8)$
$= (x - 2)(x + 4)(x - 2)$
factors are $(x - 2)(x + 4)(x - 2)$

6 $f(x) = 4x^3 - 3x^2 + 5x + k$
$f(-1) = 0$
$\Rightarrow 4(-1)^3 - 3(-1)^2 + 5(-1) + k = 0$
$\Rightarrow -4 - 3 - 5 + k = 0$
$k = 12$

7 $f(x) = x^3 + 3x^2 - 6x - 8$
$f(2) = (2)^3 + 3(2)^2 - 6(2) - 8$
$= 8 + 12 - 12 - 8$
$= 0$
$x^3 + 3x^2 - 6x - 8 = (x - 2)(x^2 + 5x + 4)$
$= (x - 2)(x + 1)(x + 4)$
$(x - 2)(x + 1)(x + 4) = 0$
$x = 2, -1, -4$

8 $f(x) = 3x^3 - kx^2 + 5x + 2$
$$f\left(\frac{1}{2}\right) = 0$$
$$\Rightarrow 3\left(\frac{1}{2}\right)^3 - k\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + 2 = 0$$
$$\Rightarrow \frac{3}{8} - \frac{1}{4}k + \frac{5}{2} + 2 = 0$$
$$\frac{1}{4}k = \frac{39}{8}$$
$$k = \frac{39}{2}$$

9 $x^3 + 5x^2 + 3x - 1 = 0$
$f(x) = x^3 + 5x^2 + 3x - 1$
$f(-1) = -1 + 5 - 3 - 1 = 0$
$x^3 + 5x^2 + 3x - 1 = (x + 1)(x^2 + 4x - 1)$
$(x + 1)(x^2 + 4x - 1) = 0$
$x = -1, x^2 + 4x - 1 = 0$
$$x = \frac{-4 \pm \sqrt{20}}{2}$$
$$= \frac{-4 \pm 2\sqrt{5}}{2}$$
$$= -2 \pm \sqrt{5}$$

10 (a) $f(x) = 4x^3 + ax^2 + bx + 3$
$f(-3) = 4(-3)^3 + a(-3)^2 + b(-3) + 3 = 0$
$-108 + 9a - 3b + 3 = 0$
$9a - 3b = 105$
$3a - b = 35$ [1]
$f(2) = 4(2)^3 + a(2)^2 + b(2) + 3$
$= 32 + 4a + 2b + 3$
$= 4a + 2b + 35$
$f(2) = 165$

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$4a + 2b + 35 = 165$
$4a + 2b = 130$
$2a + b = 65$ [2]
$[1] + [2] \Rightarrow 5a = 100, a = 20$
$40 + b = 65, b = 25$
$\therefore a = 20, b = 25$

(b) $f(x) = 4x^3 + 20x^2 + 25x + 3$
$4x^3 + 20x^2 + 25x + 3 = (x + 3)(ax^2 + bx + c)$
$= (x + 3)(4x^2 + 8x + 1)$

11 (a) $f(x) = x^4 + x^3 + ax^2 + bx + 10$
Now $x^2 - 3x + 2 = (x - 1)(x - 2)$
$\therefore x - 1$ and $x - 2$ are factor of $f(x)$
$a + b = -12$ [1]
$f(2) = 0 \Rightarrow 2^4 + 2^3 + a(2)^2 + b(2) + 10 = 0$
$4a + 2b = -34$
$2a + b = -17$ [2]
$[2] - [1] \Rightarrow a = -5$
$-5 + b = -12$
$b = -7$
$\therefore a = -5, b = -7$

(b) $f(x) = x^4 + x^3 - 5x^2 - 7x + 10$
$x^4 + x^3 - 5x^2 - 7x + 10 = (x^2 - 3x + 2)(x^2 + 4x + 5)$
Other quadratic factor is $x^2 + 4x + 5$

12 $f(x) = 36x^3 + ax^2 + bx - 2$

$$f\left(-\frac{1}{6}\right) = 0 \Rightarrow 36\left(-\frac{1}{6}\right)^3 + a\left(-\frac{1}{6}\right)^2 + b\left(-\frac{1}{6}\right) - 2 = 0$$

$$\Rightarrow \frac{1}{36}a - \frac{1}{6}b = \frac{13}{6}$$

$$f\left(\frac{2}{3}\right) = 0 \Rightarrow 36\left(\frac{2}{3}\right)^3 + a\left(\frac{2}{3}\right)^2 + b\left(\frac{2}{3}\right) - 2 = 0$$

$$\frac{4}{9}a + \frac{2}{3}b = \frac{-26}{3}$$

$\frac{1}{6}a - b = 13$ [1]

$\frac{2}{3}a + b = -13$ [2]

$[1] + [2] \Rightarrow \frac{5}{6}a = 0$

$a = 0$
$a = 0, b = -13$
$f(x) = 36x^3 - 13x - 2$
$36x^3 - 13x - 2 = (6x + 1)(3x - 2)(2x + 1)$

13 $f(x) = 4x^3 + ax^2 + bx + 3$
$f(-3) = 0$
$\Rightarrow 4(-3)^3 + a(-3)^2 + b(-3) + 3 = 0$
$\Rightarrow 9a - 3b = 105$
$3a - b = 35$ [1]
$f(1) = -12$
$\Rightarrow 4 + a + b + 3 = -12$

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$a + b = -19$ [2]
$[1] + [2] \Rightarrow 4a = 16$
$a = 4$
$4 + b = -19$
$b = -23$
$a = 4, b = -23$

14 $g(x) = 4x^4 + px^3 - 21x^2 + qx + 27$
$g(3) = 0$
$\Rightarrow 4(3)^4 + p(3)^3 - 21(3)^2 + q(3) + 27 = 0$
$27p + 3q = -162$
$9p + q = -54$ [1]
$g(-1) = 0$
$\Rightarrow 4 - p - 21 - q + 27 = 0$
$p + q = 10$ [2]
$[1] - [2] \Rightarrow 8p = -64$
$p = -8$
$-8 + q = 10$
$q = 18$
Hence $p = -8, q = 18$
$4x^4 - 8x^3 - 21x^2 + 18x + 27 = (x - 3)(x + 1)(4x^2 - 9)$
$= (x - 3)(x + 1)(2x - 3)(2x + 3)$
$a = 4, b = -9$

Try these 4.5

(a) $27x^3 - 64 = (3x)^3 - 4^3 = (3x - 4)((3x)^2 + (3x)(4) + 4^2)$
$= (3x - 4)(9x^2 + 12x + 16)$
(b) $81x^4 - 16 = (3x)^4 - 2^4 = ((3x)^2 - (2)^2)((3x)^2 + (2)^2)$
$= (3x - 2)(3x + 2)(9x^2 + 4)$
(c) $x^6 - y^6 = (x^3 - y^3)(x^3 + y^3) = (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$
$= (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)$

Review exercise 4

1 $x^6 - 64 = (x^3)^2 - (2^3)^2$
$= (x^3 - 8)(x^3 + 8)$
$= (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$

2 $32x^5 - 243$
$= (2x)^5 - 3^5$
$= (2x - 3)((2x)^4 + (2x)^3(3) + (2x)^2(3)^2 + (2x)(3)^3 + 3^4)$
$= (2x - 3)(16x^4 + 24x^3 + 36x^2 + 54x + 81)$

3 (a) $y = 6x^3 + x^2 - 5x - 2$
if $x = 1, y = 6 + 1 - 5 - 2 = 0$
$\therefore$ $x - 1$ is a factor
$\therefore$ $y = (x - 1)(6x^2 + 7x + 2)$
$= (x - 1)(3x + 1)(x + 2)$
$y = (x - 1)(3x + 1)(x + 2)$
when $x = 0, y = -2, (0, -2)$

(b) $y = 3x^3 - 2x^2 - 7x - 2$

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if $x = -1,\ y = -3 - 2 + 7 - 2 = 0$
$\therefore\ x + 1$ is a factor
$3x^3 - 2x^2 - 7x - 2 = (x + 1)(3x^2 - 5x - 2)$
$= (x + 1)(3x + 1)(x - 2)$
$y = (x + 1)(3x + 1)(x - 2)$.
when $x = 0,\ y = -2$
when $y = 0,\ x = 2, -1, -\frac{1}{3}$

(c) $y = 4x^3 + 9x^2 - 10x - 3$
if $x = 1,\ y = 4 + 9 - 10 - 3 = 0$
$\therefore\ x - 1$ is a factor
$4x^3 + 9x^2 - 10x - 3 = (x - 1)(4x^2 + 13x + 3)$
$= (x - 1)(4x + 1)(x + 3)$
$y = (x - 1)(4x + 1)(x + 3)$
when $x = 0,\ y = -3$
when $y = 0,\ x = 1, -\frac{1}{4}, -3$

4 (a) $f(x) = x^3 - 2x^2 - 4x + 8 = 0$
$f(2) = 8 - 8 - 8 + 8 = 0$
$\therefore\ x - 2$ is a factor of $f(x)$
$x^3 - 2x^2 - 4x + 8 = (x - 2)(x^2 - 4)$
$= (x - 2)(x - 2)(x + 2)$
$\therefore\ x - 2 = 0,\ x + 2 = 0$
$x = 2, -2$

(b) $f(x) = 2x^3 - 7x^2 - 10x + 24 = 0$
$f(4) = 2(4)^3 - 7(4)^2 - 10(4) + 24$
$= 0$
$\therefore\ x - 4$ is a factor of $f(x)$
$\therefore\ 2x^3 - 7x^2 - 10x + 24 = (x - 4)(2x^2 + x - 6)$
$= (x - 4)(2x - 3)(x + 2)$
$x = 4,\ x = \frac{3}{2},\ x = -2$

5 (a) $f(x) = 7x^3 - 5x^2 + 2x + 1$
$$f\left(\frac{2}{3}\right) = 7\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) + 1 = \frac{59}{27}$$

(b) $f(x) = 6x^3 + 7x + 1$
$f(-3) = 6(-3)^3 + 7(-3) + 1 = -182$

(c) $f(x) = x^4 + 2$
$$f\left(-\frac{5}{2}\right) = \left(\frac{-5}{2}\right)^4 + 2 = \frac{657}{16}$$

6 $f(x) = x^3 - ax^2 + 2ax - 8$
$f(2) = (2)^3 - a(2)^2 + 2a(2) - 8$
$= 8 - 4a + 4a - 8$
$= 0$
$\therefore\ x - 2$ is a factor of $f(x)$

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$$
\begin{array}{r|l}
 & x^2 + (-a + 2)\,x + 4 \\
x - 2 & x^3 - ax^2 + 2ax - 8 \\
 & \underline{x^3 - 2x^2} \\
 & (-a + 2)\,x^2 + 2ax - 8 \\
 & \underline{(-a + 2)\,x^2 - 2\,(-a + 2)\,x} \\
 & 4x - 8 \\
 & \underline{4x - 8} \\
 & 0
\end{array}
$$

$f(x) = (x - 2)\,(x^2 + (2 - a)\,x) + 4$

$x^2 + (2 - a)\,x + 4 = 0$

Since all roots are real $b^2 - 4ac \geq 0$

$(2 - a)^2 - 4\,(4) \geq 0$

$a^2 - 4a - 12 \geq 0$

$(a - 6)\,(a + 2) \geq 0$

$a \leq -2,\ a \geq 6$

7 (a) $f\,(x) = (3x + 2)\,(x - 1)\,(x - 2)$

$= (3x^2 - x - 2)\,(x - 2)$

$= 3x^3 - 6x^2 - x^2 + 2x - 2x + 4$

$= 3x^3 - 7x^2 + 4$

$= Ax^3 + Bx^2 + Cx + D$

$A = 3,\ B = -7,\ C = 0,\ D = 4$

(b) $f\,(-2) - 2b = 0$

$2b = f\,(-2)$

$2b = (-6 + 2)\,(-2 - 1)\,(-2 - 2)$

$2b = -48$

$b = -24$

8 (a) $f\,(x) = x^4 + 5x^3 + 4x^2 - 10x - 12$

$f\,(\sqrt{2}) = (\sqrt{2})^4 + 5\,(\sqrt{2})^3 + 4\,(\sqrt{2})^2 - 10\sqrt{2} - 12$

$= 4 + 10\sqrt{2} + 8 - 10\sqrt{2} - 12 = 0$

$\Rightarrow x - \sqrt{2}$ is a factor of $f\,(x)$

$f\,(-\sqrt{2}) = (-\sqrt{2})^4 + 5\,(-\sqrt{2})^3 + 4\,(-\sqrt{2})^2 - 10\,(-\sqrt{2}) - 12$

$= 4 - 10\sqrt{2} + 8 + 10\sqrt{2} - 12$

$= 0$

$\Rightarrow x + \sqrt{2}$ is a factor of $f\,(x)$.

A quadratic factor is $(x - \sqrt{2})\,(x + \sqrt{2}) = x^2 - 2$

$x^4 + 5x^3 + 4x^2 - 10x - 12 = (x^2 - 2)\,(x^2 + 5x + 6)$

The second quadratic factor is $x^2 + 5x + 6$

(b) $f\,(x) = x^4 + 5x^3 + 4x^2 - 10x - 12$

$f\,(-2) = (-2)^4 + 5\,(-2)^3 + 4\,(-2)^2 - 10\,(-2) - 12 = 0$

$x + 2$ is a factor of $f\,(x)$

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$$\begin{array}{r|l} & x^3 + 3x^2 - 2x - 6 \\ \hline x+2 & x^4 + 5x^3 + 4x^2 - 10x - 12 \\ & \underline{x^4 + 2x^3} \\ & 3x^3 + 4x^2 - 10x - 12 \\ & \underline{3x^3 + 6x^2} \\ & -2x^2 - 10x - 12 \\ & \underline{-2x^2 - 4x} \\ & -6x - 12 \\ & \underline{-6x - 12} \\ & 0 \end{array}$$

$\therefore$ f (x) = (x + 2) ($x^3 + 3x^2 - 2x - 6$)
= (x + 2) (x + 3) ($x^2 - 2$)
= (x + 2) (x + 3) ($x - \sqrt{2}$) ($x + \sqrt{2}$)
$\therefore$ x + 2 = 0, x + 3 = 0, $x - \sqrt{2} = 0$, $x + \sqrt{2} = 0$
x = – 2, – 3, $\sqrt{2}$, $-\sqrt{2}$

9 q (x) = $x^4 - ax^3 + bx^2 + x + 6$
q (2) = 0
$\Rightarrow (2)^4 - a(2)^3 + b(2)^2 + 2 + 6 = 0$
$\Rightarrow$ 16 – 8a + 4b + 8 = 0
– 8a + 4b = – 24
– 2a + b = – 6 [1]
q (3) = 0
$(3)^4 - a(3)^3 + b(3)^2 + 3 + 6 = 0$
81 – 27a + 9b + 9 = 0
– 27a + 9b = – 90
– 3a + b = – 10 [2]
[1] – [2] $\Rightarrow$ a = 4
b = 2
Hence a = 4, b = 2

10 (2x + 1) (x – 2) (3x + 4)
= ($2x^2 - 3x - 2$) (3x + 4)
= $6x^3 + 8x^2 - 9x^2 - 12x - 6x - 8$
= $6x^3 - x^2 - 18x - 8$
= $Ax^3 + Bx^2 + Cx + D$
A = 6, B = – 1, C = – 18, D = – 8

11 $3x^4 + Bx^3 + Cx^2 + Dx + 2 = (3x^2 + 2x + 1)(x^2 - 4x + 2)$
= $3x^4 - 12x^3 + 6x^2 + 2x^3 - 8x^2 + 4x + x^2 - 4x + 2$
= $3x^4 - 10x^3 - x^2 + 2$
$\therefore$ B = – 10, C = – 1, D = 0

12 f (x) = $ax^3 - bx^2 + 8x + 2$
f (– 2) = – 110
$\Rightarrow$ – 8a – 4b – 16 + 2 = – 110
$\Rightarrow$ 8a + 4b = 96
2a + b = 24 [1]
f (1) = a – b + 8 + 2
= a – b + 10
a – b + 10 = 13

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$a - b = 3$ [2]
[1] + [2] $\Rightarrow 3a = 27$
$a = 9$
$b = 6$
$\therefore\ f(x) = 9x^3 - 6x^2 + 8x + 2$

$$f\left(\frac{-2}{3}\right) = 9\left(\frac{-2}{3}\right)^3 - 6\left(\frac{-2}{3}\right)^2 + 8\left(\frac{-2}{3}\right) + 2$$

$$= -\frac{8}{3} - \frac{8}{3} - \frac{16}{3} + 2$$

$$= \frac{-26}{3}$$

13 $f(x) = 2x^3 - x^2 - 5x - 2.$
$f(-1) = 2(-1)^3 - (-1)^2 - 5(-1) - 2$
$= -2 - 1 + 5 - 2 = 0$
$\Rightarrow x + 1$ is a factor of f (x)
$2x^3 - x^2 - 5x - 2 = (x + 1)(2x^2 - 3x - 2)$
$= (x + 1)(2x + 1)(x - 2)$

14 $f(x) = 6x^3 - 5x^2 - 13x - 2$
$f(-1) = -6 - 5 + 13 - 2 = 0$
$x + 1$ is a factor of f (x)
$6x^3 - 5x^2 - 13x - 2 = (x + 1)(6x^2 - 11x - 2)$
$= (x + 1)(6x + 1)(x - 2)$
$\therefore\ x + 1 = 0,\ 6x + 1 = 0,\ x - 2 = 0$

$$x = -1,\ -\frac{1}{6},\ 2$$

15 $f(x) = x^4 + x^3 - 11x^2 - 27x - 36$
$f(-3) = 81 - 27 - 99 + 81 - 36$
$= 0.$
$\therefore\ x + 3$ is a factor of f (x)

$$\begin{array}{r|l} & x^3 - 2x^2 - 5x - 12 \\ \hline x + 3 & x^4 + x^3 - 11x^2 - 27x - 36 \\ & \underline{x^4 + 3x^3} \\ & \quad -2x^3 - 11x^2 - 27x - 36 \\ & \quad \underline{-2x^3 - 6x^2} \\ & \qquad -5x^2 - 27x - 36 \\ & \qquad \underline{-5x^2 - 15x} \\ & \qquad\quad -12x - 36 \\ & \qquad\quad \underline{-12x - 36} \\ & \qquad\qquad 0 \end{array}$$

$f(x) = (x + 3)(x^3 - 2x^2 - 5x - 12)$
$= (x + 3)(x - 4)(x^2 + 2x + 3)$
$\therefore\ x + 3 = 0,\ x - 4 = 0$
$x = -3,\ x = 4$
$x^2 + 2x + 3 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 12}}{2}$$, so no more real solutions

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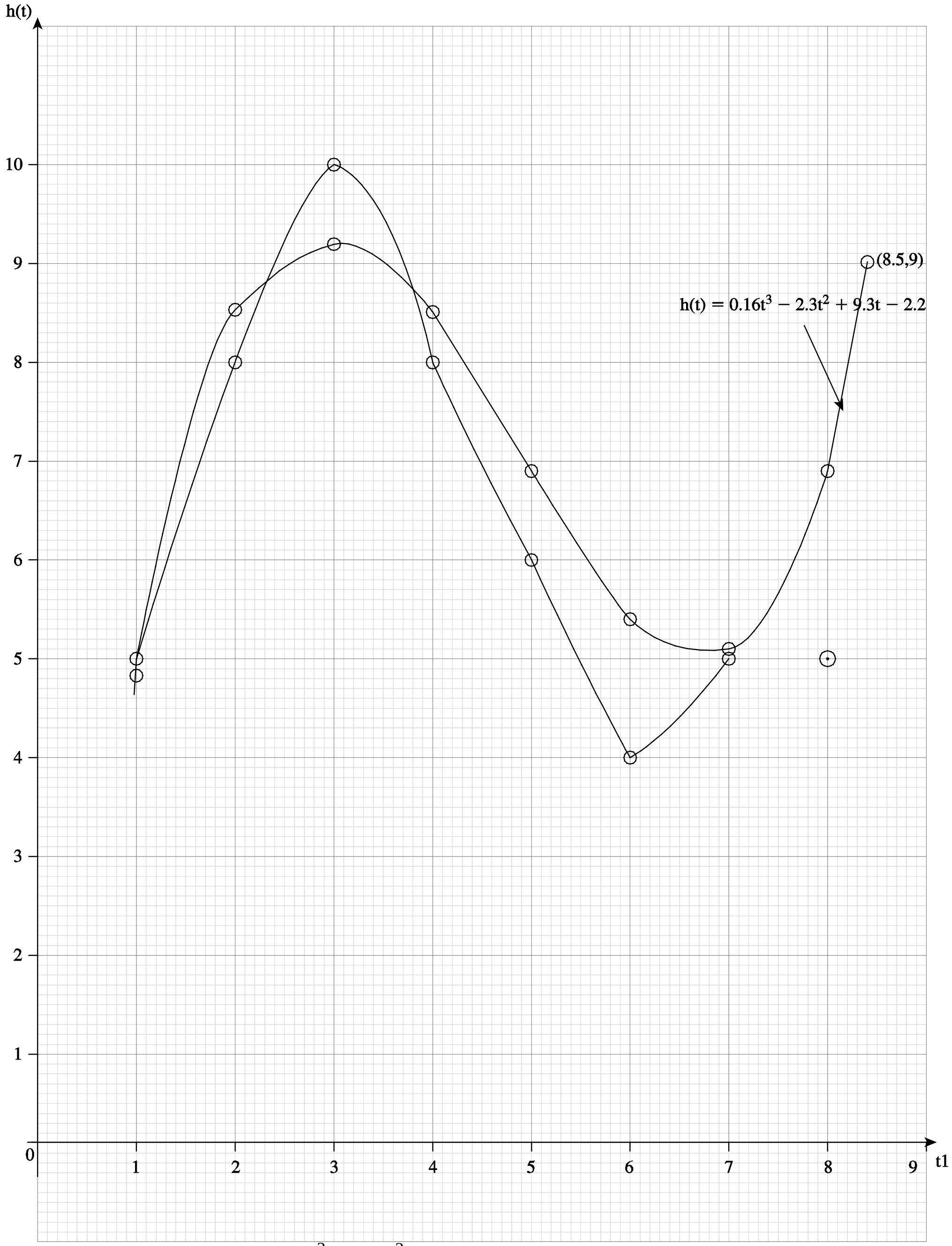
16 (a)



(b) $h(t) = 0.16t^3 - 2.3t^2 + 9.3t - 2.2$

1971 to 1980, $t = 6$

$\therefore h(6) = 0.16(6)^3 - 2.3(6)^2 + 9.3(6) - 2.2$

$= 5.36$

(c)

t	1	2	3	4	5	6	7	8
h (t)	4.96	8.48	9.32	8.44	6.8	5.36	5.08	6.92

(d) Only for a section of the data. The model may need refining

(e) The model predicts approximately 9 hurricanes, the model does not support 5

17 (a) Let one side of the square be x cm

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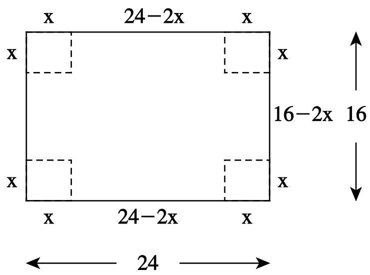

$V = (24 - 2x)(16 - 2x)(x)$

$= 384x - 80x^2 + 4x^3$

(b) $V = 512$

$4x^3 - 80x^2 + 384x - 512 = 0$

if $x = 4$, $4(4)^3 - 80(4)^2 + 384(4) - 512 = 256 - 1280 + 1536 - 512$

$= 0$

$4x^3 - 80x^2 + 384x - 512 = (x - 4)(4x^2 - 64x - 128)$

$= 4(x - 4)(x^2 - 16x - 32)$

Ht. $x = 4$, $x^2 - 16x - 32 = 0$

$$x = \frac{16 \pm \sqrt{384}}{2} = \frac{16 \pm 19.6}{2}$$

$= 17.8, - 1.8$

So the only possible size for the corner cut out is $x = 4$

18 (a) $f(x) = 2x^3 - 7x^2 - 10x + 24$

$= (x - 4)(2x^2 + x - 6)$

$= (x - 4)(2x - 3)(x + 2)$

(b) $x = 4, \frac{3}{2}, -2$

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