Chapter 1 Reasoning and Logic

Try these 1.1

(a) Today is not Sunday.
(b) The music is not loud.
(c) Lorraine does not teach physics.

Try these 1.2

(a) (i) Let p be 'the measuring cylinder is full'
q 'the beaker is empty'
$p \vee q$
(ii) b 'breakfast is at the house'
c 'the coffee is hot'
$b \wedge c$
(b) (i) 'The air-condition unit is not broken'
(ii) 'December 25th is not Christmas'

Try these 1.3

(a) Paris is in France is true
Rome is in Italy is also true
$\therefore$ Paris is in France if and only if Rome is in Italy is true
(b) $8 - 4 = 12$ is false
$30 + 24 = 64$ is false
$\therefore 8 - 4 \leftrightarrow 30 + 24 = 64$ is true
(c) Port of Spain is the capital of Trinidad and Tobago is true $\frac{4}{6} = \frac{1}{2}$ is false
$\therefore$ Port of Spain is the capital of Trinidad and Tobago if and only if $\frac{4}{6} = \frac{1}{2}$ is false

Exercise 1A

1 (a) Yes
(b) No
(c) Yes
(d) No
(e) Yes

2 (a) Let m: It is May
c: CAPE examination begins
$m \wedge c$
(b) S: takes Spanish
A: takes additional mathematics
$s \vee a$
(c) l: the music is loud
r: ryan is speaking
$l \wedge r$

(d) m: drive to Montego Bay
b: go by bus
p: go by plane
$\sim m \wedge (b \vee p)$

3 Conjunction

4 (a) Alvin is tall or Sintra is not short
(b) Alvin is not tall or Sintra is not short
(c) Alvin is tall and Sintra is not short
(d) Alvin is not tall and Sintra is not short

5 (a) 4 is not a complete square
(b) the iPod is not white
(c) Robin likes to work overtime
(d) $\sqrt{7}$ is rational.

6 (a) $\sim (s \vee p) = \sim s \wedge \sim p$
Statistics is easy and probability is difficult
(b) $\sim (s \wedge \sim p) = \sim s \vee p$
Statistics or probability is easy
(c) $\sim (\sim s \wedge p) = s \vee \sim p$: Statistics or probability is difficult

7 (a) $\sim (p \vee q)$
$= \sim (p \vee q)$
$= \sim p \wedge \sim q$
Jamaica is not beautiful and Watson does not like jerk chicken
(b) $\sim (p \wedge \sim q)$
$= \sim p \vee q$
Jamaica is not beautiful or Watson likes jerk chicken
(c) $\sim (\sim p \wedge \sim q)$
$= p \vee q$
Jamaica is beautiful or Watson likes jerk chicken

8 (a) $x \wedge y$
(b) $\sim (x \vee y)$
(c) $\sim (x \wedge y)$

9

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Only if both ‘p’ and ‘q’ are false, the disjunction $p \vee q$ is false

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r	c	$r \wedge c$
T	T	T
T	F	F

F	T	F
F	F	F

The conjunction r ∧ c is true only if both r is true and c is true

Exercise 1B

1 The music is not good if I don't dance

2 (a) I will go running and the sun is shining
(b) The sun is not shining
(c) Either the sun is shining or I am going running

3 (a) If the teacher will not write then the board will not be clean
(b) Chris will not pass calculus if he does not study

4

p	q	q ∨ p	~ (q ∨ p)	p → ~ (q ∨ p)
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	T

5 (p ∨ ~ q) ∧ (~ p ∧ q)
= (p ∨ ~ q) ∧ ~ (p ∨ ~ q)
= F
Let A = (p ∨ ~ q)
A ∧ ~ A = F
∴ (p ∨ ~ q) ∧ (~ p ∧ q) is a contradiction since the statement is always false

6 (~ (x ∨ y)) ∨ ((~ x) ∧ y)
= (~ x ∧ ~ y) ∨ (~ x ∧ y)
= ~ x ∧ (~ y ∨ y)
= ~ x ∧ T
= ~ x

7 Contrapositive: The sun is not shining whenever the Trinidad and Tobago cricket team loses
Converse: The sun is shining whenever the Trinidad and Tobago cricket team wins
Inverse: The Trinidad and Tobago cricket team loses whenever the sun is not shining

8

p	q	~ q	(p ∨ ~ q)	p ∧ q	(p ∨ ~ q) ⇒ (p ∧ q)
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

9 (a)

p	q	~ p	~ q	(~ p ∨ ~ q)	~ (~ p ∨ ~ q)
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

(b)

p	q	~ p	~ q	(~ p ∧ ~ q)	~ (~ p ∧ ~ q)
T	T	F	F	F	T

T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

(c)

p	q	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	F
F	T	T	F	T	F
F	F	F	F	T	F

(d)

p	q	r	$(q \vee r)$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

10 s ‘access the staff room’
m ‘member of staff’
f ‘first year student’
$s \wedge (m \vee \sim f)$

11 w ‘watch the movie’
o ‘under 13 years old’
a ‘accompanied by an adult’
$\sim w \wedge (o \wedge \sim a)$

12 (a) I did not buy peanut butter this week
(b) I either bought peanut butter this week or made peanut punch on Saturday
(c) I bought peanut butter this week and made peanut punch on Saturday
(d) We can simplify the expression:
$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$
$= T \wedge (\sim p \vee q)$
$= \sim p \vee q$
I did not buy peanut butter this week or I made peanut punch on Saturday

13 (a) $\sim p$
(b) $p \wedge \sim q$
(c) $p \rightarrow q$
(d) $q \rightarrow p$

14 (a)

p	q	$\sim q$	$(p \vee \sim q)$	$(p \vee \sim q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

(b)

p	q	$(p \vee q)$	$(p \wedge q)$	$(p \vee q) \rightarrow (p \wedge q)$

T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

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p	q	$(p \rightarrow q)$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

For the same combinations of p and q, the output for $(p \rightarrow q) \wedge (q \rightarrow p)$ and $p \leftrightarrow q$ are the same therefore the statements are logically equivalent

16 $\sim (p \vee (\sim p \wedge q))$
$= \sim ((p \vee \sim p) \wedge (p \vee q))$
$= \sim (T \wedge (p \vee q))$
$= \sim (p \vee q)$
$= \sim p \wedge \sim q$

17 (a) Either the mango is sweet or yellow
(b) The mango is not yellow
(c) The mango is both sweet and yellow

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p	q	$\sim q$	$(p \leftrightarrow \sim q)$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	F

This is not a contradiction

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p	q	$\sim p$	$\sim q$	$(p \vee \sim q)$	$(\sim p \wedge q)$	$(p \vee \sim q) \rightarrow (\sim p \wedge q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	F	F

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p	q	$\sim p$	$\sim q$	$\sim (p \vee q)$	$(\sim p \wedge \sim q)$	$\sim (p \vee q) \leftrightarrow (\sim p \wedge \sim q)$
T	T	F	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

The statement is always true

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p	q	$(p \wedge q)$	$p \wedge q \rightarrow p$
T	T	T	T

F	T	F	T
T	F	F	T
F	F	F	T

Since the statement is always true, the statement is a tautology

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p	q	$p \vee q$	$\sim q$	$(p \vee q) \wedge \sim q$	$\sim (p \wedge q)$	$(p \wedge \sim q)$	$\sim (p \wedge q) \wedge (p \wedge \sim q)$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	F	F	F	F	F	F
T	F	T	T	T	T	T	T	T
F	T	T	F	F	T	F	F	T
F	F	F	T	F	T	F	F	F

$(p \vee q) \wedge \sim q \not\equiv (p \vee q) \wedge \sim (p \wedge q)$

$(p \vee q) \wedge \sim (p \wedge q) \not\equiv \sim (p \wedge q) \wedge (p \wedge \sim q)$

But $(p \vee q) \wedge \sim q \equiv \sim (p \wedge q) \wedge (p \wedge \sim q)$

23 (a) Deepak works out everyday and has muscles

Deepak doesn't work out everyday and does not have muscles

(b) (i) $\bar{q} \rightarrow \bar{p}$

(ii) $\bar{p} \rightarrow \bar{q}$

(iii) $q \rightarrow p$

24 (a) $p \rightarrow q$

(b) $p \wedge q$

(c)

p	q	$p \rightarrow q$	$p \wedge q$
T	F	F	F

25 $\sim ((p \wedge q) \wedge (p \vee \sim q))$

$= \sim ((p \wedge q) \wedge p) \vee ((p \wedge q) \wedge \sim q)$

$= \sim ((p \wedge q) \vee F)$

$= \sim (p \wedge q)$