Chapter 12 Limits and Continuity

Try these 12.1

(a) $\lim_{x\to 2} (4x^2+6x+1) = 4(2)^2+6(2)+1 = 16+12+1 = 29$

(b) $\lim_{x\to 1} \left(\frac{x^2+1}{x-4}\right) = \frac{1+1}{1-4} = -\frac{2}{3}$

(c) $\lim_{t\to 0} \sqrt{\frac{t+1}{t+9}} = \sqrt{\frac{0+1}{0+9}} = \frac{1}{3}$

(d) $\lim_{t\to 4} (3t+1)^3 = (3(4)+1)^3 = 2197$

(e) $\lim_{t\to 4} (t+1)^2(4t-2) = (4+1)^2(4(4)-2) = 350$

(f) $\lim_{x\to 0} (3x-4)(2x^2+7) = (3(0)-4)(2(0)^2+7) = (-4)(7) = -28$

(g) $\lim_{x\to -2} (4x^3-3x^2+5) = 4(-2)^3-3(-2)^2+5 = -39$

(h) $\lim_{x\to -1} \left(\frac{x+1}{2x+3}\right) = \frac{-1+1}{2(-1)+3} = 0$

(i) $\lim_{x\to 0} \sqrt{\frac{x^2+2x+1}{x+2}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

(j) $\lim_{x\to 2} \sqrt{\frac{(x+1)^3}{4x-2}} = \sqrt{\frac{(2+1)^3}{4(2)-2}} = \sqrt{\frac{27}{6}} = \frac{3}{\sqrt{2}} = \frac{3}{2}\sqrt{2}$

Try these 12.2

(a) $\lim_{x\to 1} \frac{x^3-1}{x-1}$

$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$, factoring x^3-1

$= \lim_{x\to 1} (x^2+x+1)$

$= (1)^2+1+1$

$= 3$

(b) $\lim_{x\to -1} \frac{x^3+1}{x+1}$

$= \lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}}$

$= \lim_{x\to -1} (x^2-x+1)$

$= (-1)^2-(-1)+1$

$= 3$

(c) $\lim_{x\to 0} \left(\frac{x^2+x}{x}\right)$

$$= \lim_{x\to 0} (x+1)$$
$$= 0+1$$
$$= 1$$

Try these 12.3

(a) $\lim_{x\to 25} \dfrac{5-\sqrt{x}}{25-x}$

$$= \lim_{x\to 25} \frac{5-\sqrt{x}}{25-x} \times \frac{5+\sqrt{x}}{5+\sqrt{x}}$$

$$= \lim_{x\to 25} \frac{\cancel{(25-x)}}{\cancel{(25-x)}(5-\sqrt{x})}$$

$$= \lim_{x\to 25} \frac{1}{5+\sqrt{x}} \qquad \sqrt{\ }$$

$$= \frac{1}{5+\sqrt{25}} = \frac{1}{10}$$

(b) $\lim_{x\to 19} \dfrac{x-19}{5-\sqrt{x+6}}$

$$= \lim_{x\to 19} \frac{x-19}{5-\sqrt{x+6}} \times \frac{5+\sqrt{x+6}}{5+\sqrt{x+6}}$$

$$= \lim_{x\to 19} \frac{(x-19)(5+\sqrt{x+6})}{25-(x+6)}$$

$$= \lim_{x\to 19} \frac{(x-19)(5+\sqrt{x+6})}{19-x}$$

$$= \lim_{x\to 19} -(5+\sqrt{x+6})$$

$$= -(5+\sqrt{25})$$

$$= -10$$

(c) $\lim_{x\to 5} \dfrac{x-5}{\sqrt{x-5}}$

$$= \lim_{x\to 5} \frac{x-5}{\sqrt{x-5}} \times \frac{\sqrt{x-5}}{\sqrt{x-5}}$$

$$= \lim_{x\to 5} \frac{\cancel{(x-5)}\sqrt{x-5}}{\cancel{(x-5)}}$$

$$= \lim_{x\to 5} \sqrt{x-5}$$

$$= \sqrt{5-5} = 0$$

Try these 12.4

(a) $\lim_{x\to \infty} \dfrac{x^2+6x+9}{2x^2+7x+8}$

$$= \lim_{x\to\infty} \frac{1+\frac{6}{x}+\frac{9}{x^2}}{2+\frac{7}{x}+\frac{8}{x^2}}$$

$$= \frac{1}{2}$$

(b) $$\lim_{x\to\infty} \frac{x^3+7x^2+2}{3x^3+6x+1}$$

$$= \lim_{x\to\infty} \frac{1+\frac{7}{x}+\frac{2}{x^3}}{3+\frac{6}{x^2}+\frac{1}{x^3}}$$

$$= \frac{1}{3}$$

Exercise 12A

1 $\lim_{x\to 2} (x^2+4x+5) = 2^2+4(2)+5$

$= 17$

2 $\lim_{x\to 0} 3^x = 3^o = 1$

3 $\lim_{x\to 0} \frac{3x+2}{4x-1} = \frac{3(0)+2}{4(0)-1}$

$= -2$

4 $\lim_{x\to 2} \frac{3x^2+5x+2}{x-4} = \frac{3(2)^2+5(2)+2}{2-4}$

$= \frac{24}{-2} = -12$

5 $\lim_{x\to -1} \frac{4x^2-3x+2}{x^2+x+2} = \frac{4(-1)^2-3(-1)+2}{(-1)^2+(-1)+2}$

$= \frac{9}{2}$

6 $\lim_{x\to 1} (4x+3)^2 = (4+3)^2 = 49$

7 $\lim_{x\to 5} \frac{\cancel{(x-5)}(x+2)}{\cancel{x-5}} = \lim_{x\to 5} (x+2)$

$= 5+2 = 7$

8 $\lim_{x\to 3} \frac{\cancel{(x-3)}(2x+1)}{\cancel{(x-3)}} = \lim_{x\to 3} (2x+1)$

$= 2(3)+1 = 7$

9 $\lim_{x\to 0} \left(\frac{3x^3-4x^2}{x^2}\right) = \lim_{x\to 0} \frac{x^2(3x-4)}{x^2}$

$= \lim_{x\to 0} (3x-4)$

$= 3(0)-4$

$= -4$

10 $\lim_{x\to -2} \frac{x^2+5x+6}{x+2} = \lim_{x\to -2} \frac{(x+3)\cancel{(x+2)}}{\cancel{x+2}}$

$= \lim_{x\to -2} (x+3)$

$= -2+3 = 1$

11 $\lim_{x\to -5} \frac{x^2+9x+20}{x+5} = \lim_{x\to -5} \frac{(x+4)\cancel{(x+5)}}{\cancel{x+5}}$

$= \lim_{x\to -5} (x+4)$

$= -5+4 = -1$

12 $\lim_{x\to -2} \frac{x^4-16}{x+2} = \lim_{x\to -2} \frac{(x-2)\cancel{(x+2)}(x^2+4)}{\cancel{x+2}}$

$= \lim_{x\to -2} (x-2)(x^2+4)$

$= (-2-2)((-2)^2+4)$

$= -32$

13 $\lim_{x\to 5} \frac{\sqrt{x+11}-4}{x-5} = \lim_{x\to 5} \frac{\sqrt{x+11}-4}{x-5} \times \frac{\sqrt{x+11}+4}{\sqrt{x+11}+4}$

$= \lim_{x\to 5} \frac{x+11-16}{(x-5)(\sqrt{x+11}+4)}$

$= \lim_{x\to 5} \frac{\cancel{x-5}}{\cancel{x-5}(\sqrt{x+11}+4)}$

$= \lim_{x\to 5} \frac{1}{\sqrt{x+11}+4}$

$= \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \frac{1}{8}$

14 $\lim_{x\to -2} \frac{x+2}{\sqrt{x+6}-2} = \lim_{x\to -2} \frac{x+2}{\sqrt{x+6}-2} \times \frac{\sqrt{x+6}+2}{\sqrt{x+6}+2}$

$= \lim_{x\to -2} \frac{(x+2)[\sqrt{x+6}+2]}{x+6-4}$

$= \lim_{x\to -2} \frac{\cancel{(x+2)}(\sqrt{x+6}+2)}{\cancel{x+2}}$

$= \lim_{x\to -2} (\sqrt{x+6}+2)$

$= \sqrt{-2+6}+2$

$= \sqrt{4}+2 = 2+2 = 4$

15 $\lim_{x\to 16} \left(\frac{x-16}{\sqrt{x}-4}\right) = \lim_{x\to 16} \frac{x-16}{\sqrt{x}-4} \times \frac{\sqrt{x}+4}{\sqrt{x}+4}$

$= \lim_{x\to 16} \frac{\cancel{(x-16)}(\sqrt{x}+4)}{\cancel{x-16}}$

$= \lim_{x\to 16} (\sqrt{x}+4) = \sqrt{16}+4$

$= 4+4 = 8$

16 $\lim_{x\to\infty}\frac{3x+1}{2x-1}=\lim_{x\to\infty}\frac{3+\frac{1}{x}}{2-\frac{1}{x}}=\frac{2}{2}$

17 $\lim_{x\to\infty}\frac{6x-5}{4x-1}=\lim_{x\to\infty}\frac{6-\frac{5}{x}}{4-\frac{1}{x}}=\frac{6}{4}=\frac{3}{2}$

18 $\lim_{x\to\infty}\frac{x^2+3x+1}{x^2-2x-1}=\lim_{x\to\infty}\frac{1+\frac{3}{x}+\frac{1}{x^2}}{1-\frac{2}{x}-\frac{1}{x^2}}=1$

19 $\lim_{x\to\infty}\frac{x^3-6x+1}{2x^2-1}=\lim_{x\to\infty}\frac{1-\frac{6}{x^2}+\frac{1}{x^3}}{\frac{2}{x}-\frac{1}{x^3}}$

$=\frac{1}{0}\to\infty$

Try these 12.5

(a) $\lim_{\theta\to 0}\frac{\sin 8\theta}{\theta}=\lim_{\theta\to 0}\frac{8\sin 8\theta}{8\theta}$

$=8\lim_{\theta\to 0}\frac{\sin 8\theta}{8\theta}$

$=8(1)=8$

(b) $\lim_{\theta\to 0}\left(\frac{\tan 7\theta}{\theta}\right)=\lim_{\theta\to 0}\left(\frac{\sin 7\theta}{\theta\cos 7\theta}\right)$

$=\lim_{\theta\to 0}\left(\frac{7\sin 7\theta}{7\theta}\right)\lim_{\theta\to 0}\frac{1}{\cos 7\theta}$

$=(7)(1)$

$=7$

(c) $\lim_{\theta\to 0}\frac{\sin 6\theta}{\sin 5\theta}=\lim_{\theta\to 0}\left(\frac{\frac{\sin 6\theta}{\theta}}{\frac{\sin 5\theta}{\theta}}\right)$

$=\frac{\lim_{\theta\to 0}\left(\frac{6\sin 6\theta}{6\theta}\right)}{\lim_{\theta\to 0}\left(\frac{5\sin 5\theta}{5\theta}\right)}$

$=\frac{6(1)}{5(1)}$

$=\frac{6}{5}$

Exercise 12B

1 $\lim_{x\to 1}\frac{4x^3+5}{x-3} = \frac{4+5}{1-3} = \frac{-9}{2}$

2 $\lim_{x\to 0}(4x+2)^5 = (4(0)+2)^5$

$= 2^5 = 32$

3 $\lim_{x\to 2}\sqrt{4x+2} = \sqrt{4(2)+2}$

$= \sqrt{10}$

4 $\lim_{x\to 1}(x^2-2x+1)^6 = ((1)^2-2(1)+1)^6$

$= 0$

5 5. $\lim_{x\to -1}(4x+3)^3(2x+1)^2 = (4(-1)+3)^3(2(-1)+1)^2$

$= (-1)^3(-1)^2$

$= -1$

6 $\lim_{x\to 0}\frac{(6x-2)^3}{(3x+1)^2} = \frac{(6(0)-2)^3}{(3(0)+1)^2}$

$= -8$

7 $\lim_{x\to -3}\frac{x^2-3}{2x+3} = \frac{(-3)^2-3}{2(-3)+3}$

$= \frac{6}{-3} = -2$

8 $\lim_{x\to -\frac{1}{2}}\frac{2x^2+5x+2}{2x+1} = \lim_{x\to -\frac{1}{2}}\frac{\cancel{(2x+1)}(x+2)}{\cancel{2x+1}}$

$= \lim_{x\to -\frac{1}{2}}(x+2)$

$= -\frac{1}{2}+2 = \frac{3}{2}$

9 $\lim_{x\to -2}\frac{x^5+32}{x+2} = \lim_{x\to -2}\frac{\cancel{(x+2)}(x^4-2x^3+4x^2-8x+16)}{\cancel{x+2}}$

$= \lim_{x\to -2}(x^4-2x^3+4x^2-8x+16)$

$= (-2)^4-2(-2)^3+4(-2)^2-8(-2)+16$

$= 16+16+16+16+16 = 80$

10 $\lim_{x\to 5}\frac{x^2+2x-35}{x^2-10x+25} = \lim_{x\to 5}\frac{\cancel{(x-5)}(x+7)}{\cancel{(x-5)}(x-5)}$

$= \lim_{x\to 5}\frac{x+7}{x-5}$

$= \frac{5+7}{5-5} = \frac{12}{0} = \infty$

11 $\lim_{x\to -1}\frac{x^2+x-2}{x^2-1} = \lim_{x\to -1}\frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)}$

$= \lim_{x\to -1}\frac{x+2}{x+1}$

$$= \frac{-1+2}{-1+1} = \frac{3}{0} = \infty$$

12 $$\lim_{x \to 1} \frac{2x^3 + x^2 - 2x - 1}{3x^2 - 2x - 1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(2x^2 + 3x + 1)}{(3x+1)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{2x^2 + 3x + 1}{3x + 1}$$

$$= \frac{2+3+1}{3+1} = \frac{6}{4} = \frac{3}{2}$$

13 $$\lim_{x \to \frac{1}{3}} \frac{27x^3 - 1}{3x - 1} = \lim_{x \to 1/3} \frac{\cancel{(3x-1)}(9x^2 + 3x + 1)}{\cancel{3x-1}}$$

$$= \lim_{x \to \frac{1}{3}} (9x^2 + 3x + 1)$$

$$= 9\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 1$$

$$= 1 + 1 + 1 = 3$$

14 $$\lim_{x \to 3} \frac{x^5 - 243}{x - 3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x^4 + 3x^3 + 9x^2 + 27x + 81)}{\cancel{x-3}}$$

$$= \lim_{x \to 3} (x^4 + 3x^3 + 9x^2 + 27x + 81)$$

$$= (3)^4 + 3(3)^3 + 9(3)^2 + 27(3) + 81$$

$$= 405$$

15 $$\lim_{x \to 9} \frac{9 - x}{\sqrt{x} - 3} = \lim_{x \to 9} \frac{9 - x}{\sqrt{x} - 3} \times \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

$$= \lim_{x \to 9} \frac{-\cancel{(x-9)}(\sqrt{x} + 3)}{\cancel{x-9}}$$

$$= \lim_{x \to 9} -(\sqrt{x} + 3)$$

$$= -(\sqrt{9} + 3) = -6$$

16 $$\lim_{x \to 10} \frac{\sqrt{5} - \sqrt{x-5}}{10 - x} = \lim_{x \to 10} \frac{\sqrt{5} - \sqrt{x-5}}{10 - x} \times \frac{\sqrt{5} + \sqrt{x-5}}{\sqrt{5} + \sqrt{x-5}}$$

$$= \lim_{x \to 0} \frac{5 - (x-5)}{(10 - x)(\sqrt{5} + \sqrt{x-5})}$$

$$= \lim_{x \to 10} \frac{\cancel{10-x}}{\cancel{(10-x)}\sqrt{5} + \sqrt{x-5}}$$

$$= \lim_{x \to 10} \frac{1}{\sqrt{5} + \sqrt{x-5}}$$

$$= \frac{1}{\sqrt{5} + \sqrt{5}} = \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{10}$$

17 $$\lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x} = \lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x} \times \frac{2 + \sqrt{x}}{2 + \sqrt{x}}$$

$$= \lim_{x \to 4} \frac{\cancel{4-x}}{\cancel{(4-x)}(2 + \sqrt{x})}$$

$$= \lim_{x\to4} \frac{1}{(2+\sqrt{x})}$$

$$= \frac{1}{2+\sqrt{4}} = \frac{1}{2+2} = \frac{1}{4}$$

18 $$\lim_{x\to1} \frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} = \lim_{x\to1} \frac{(\sqrt{3+x}-\sqrt{5-x})(\sqrt{3+x}+\sqrt{5-x})}{(x^2-1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$= \lim_{x\to1} \frac{(3+x)-(5-x)}{(x^2-1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$= \lim_{x\to1} \frac{-2+2x}{(x-1)(x+1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$= \lim_{x\to1} \frac{2\cancel{(x-1)}}{\cancel{(x-1)}(x+1)(\sqrt{3+x}+\sqrt{x-5})}$$

$$= \lim_{x\to1} \frac{2}{(x+1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$= \frac{2}{2(\sqrt{4}+\sqrt{4})} = \frac{2}{2(4)} = \frac{1}{4}$$

19 $$\lim_{x\to0} \frac{x^2}{\sqrt{x^2+12}-\sqrt{12}} = \lim_{x\to0} \frac{x^2}{\sqrt{x^2+12}-\sqrt{12}} \times \frac{\sqrt{x^2+12}+\sqrt{12}}{\sqrt{x^2+12}+\sqrt{12}}$$

$$= \lim_{x\to0} \frac{\cancel{x^2}\left[\sqrt{x^2+12}+\sqrt{12}\right]}{\cancel{x^2}+\cancel{12-12}}$$

$$= \lim_{x\to0} (\sqrt{x^2+12}+\sqrt{12})$$

$$= \sqrt{12}+\sqrt{12} = 2\sqrt{4\times3} = 4\sqrt{3}$$

20 $$\lim_{\theta\to0} \frac{\sin5\theta}{\theta} = \lim_{\theta\to0} \frac{5\sin5\theta}{5\theta}$$

$$= 5\lim_{\theta\to0} \frac{\sin5\theta}{5\theta}$$

$$= 5(1) = 5$$

21 $$\lim_{\theta\to0} \frac{\sin3\theta}{5\theta} = \lim_{\theta\to0} \frac{3\sin3\theta}{5(3\theta)}$$

$$= \frac{3}{5}\lim_{\theta\to0} \frac{\sin3\theta}{3\theta}$$

$$= \frac{3}{5}(1) = \frac{3}{5}$$

22 $$\lim_{\theta\to0} \frac{\sin2\theta}{3\theta} = \lim_{\theta\to0} \frac{2}{3}\frac{\sin2\theta}{2\theta}$$

$$= \frac{2}{3}\lim_{\theta\to0} \frac{\sin2\theta}{2\theta}$$

$$= \frac{2}{3}(1) = \frac{2}{3}$$

23 $\lim_{\theta \to 0} \frac{\sin 3\theta}{\sin 5\theta} = \lim_{\theta \to 0} \left[\frac{\left(\frac{\sin 3\theta}{\theta}\right)}{\frac{\sin 5\theta}{\theta}} \right]$

$= \frac{\lim_{\theta \to 0}\left(\frac{\sin 3\theta}{\theta}\right)}{\lim_{\theta \to 0}\left(\frac{\sin 5\theta}{\theta}\right)}$

$= \frac{\lim_{\theta \to 0} 3\left(\frac{\sin 3\theta}{3\theta}\right)}{\lim_{\theta \to 0} 5\left(\frac{\sin 5\theta}{5\theta}\right)}$

$= \frac{3}{5}$

24 $\lim_{\theta \to 0} \left(\frac{\sin 8\theta}{\sin 4\theta}\right) = \lim_{\theta \to 0} \frac{\left(\frac{8\sin 8\theta}{8\theta}\right)}{\left(\frac{4\sin 4\theta}{4\theta}\right)}$

$= \frac{8\lim_{\theta \to 0}\left(\frac{\sin 8\theta}{8\theta}\right)}{4\lim_{\theta \to 0}\left(\frac{\sin 4\theta}{4\theta}\right)}$

$= \frac{8}{4} = 2$

25 $\lim_{\theta \to 0} \frac{\cos 2\theta - 1}{2\theta} = 0$

26 $\lim_{\theta \to 0} \left(\frac{1 - \cos 2\theta}{2\theta}\right) = \lim_{\theta \to 0} - \frac{(\cos 2\theta - 1)}{2\theta} = 0$

27 $\lim_{\theta \to 0} \frac{\tan 2\theta}{2\theta} = \lim_{\theta \to 0} \left(\frac{\sin 2\theta}{\cos 2\theta}\right)\left(\frac{1}{2\theta}\right)$

$= \lim_{\theta \to 0} \left(\frac{\sin 2\theta}{2\theta}\right) \times \lim_{\theta \to 0} \left(\frac{1}{\cos 2\theta}\right)$

$= (1)\ (1) = 1$

28 $\lim_{\theta \to 0} \left(\frac{\tan 6\theta}{4\theta}\right) = \lim_{\theta \to 0} \left(\frac{\sin 6\theta}{\cos 6\theta}\right)\left(\frac{1}{4\theta}\right)$

$= \lim_{\theta \to 0} \left(\frac{\sin 6\theta}{4\theta}\right) \lim_{\theta \to 0} \left(\frac{1}{\cos 6\theta}\right)$

$= \lim_{\theta \to 0} \frac{6\sin 6\theta}{4(6\theta)} \lim_{\theta \to 0} \frac{1}{\cos 6\theta}$

$= \left(\frac{6}{4}\right)(1)(1)$

$= \frac{3}{2}$

29 $\lim_{x\to 0}\frac{x}{\sin 3x}=\lim_{x\to 0}\left(\frac{\frac{x}{x}}{\frac{\sin 3x}{x}}\right)$

$=\frac{\lim_{x\to 0}\left(\frac{x}{x}\right)}{\lim_{x\to 0}3\left(\frac{\sin 3x}{3x}\right)}$

$=\frac{1}{3}$

30 $\lim_{x\to 0}\frac{x}{\tan\ x}=\lim_{x\to 0}\left(\frac{\frac{x}{x}}{\frac{\tan x}{x}}\right)$

$=\frac{\lim_{x\to 0}\left(\frac{x}{x}\right)}{\lim_{x\to 0}\left(\frac{\tan x}{x}\right)}$

$=\frac{\lim_{x\to 0}1}{\lim_{x\to 0}\left(\frac{\sin x}{x\cos x}\right)}$

$=\frac{1}{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\lim_{x\to 0}\frac{1}{\cos x}}$

$=\frac{1}{(1)(1)}=1$

31 $\lim_{x\to\infty}\left(\frac{4x^4+3x^2+x-2}{x^4-1}\right)=\lim_{x\to\infty}\left(\frac{4+\frac{3}{x^2}+\frac{1}{x^3}-\frac{2}{x^4}}{1-\frac{1}{x^4}}\right)$

$=\frac{4}{1}=4$

32 $\lim_{x\to\infty}\left(\frac{3x^2}{x^4-1}\right)=\lim_{x\to\infty}\left(\frac{\frac{3}{x^2}}{1-\frac{1}{x^4}}\right)$

$=\frac{0}{1}=0$

33 (a) $\lim_{x\to 2}3f(x)=10\Rightarrow\lim_{x\to 2}f(x)=\frac{10}{3}$

$\lim_{x\to 2}\left[f(x)+2x^2\right]=\lim_{x\to 2}f(x)+2\lim_{x\to 2}x^2=\frac{10}{3}+2(2)^2=\frac{34}{3}$

(b) $x^2+7x+12=0$

$(x+3)(x+4)=0$

$x=-3,-4$

the function is not continuous when x = –3, –4

34 (a) $\frac{\sqrt{x-6}-1}{7-x} = \frac{(\sqrt{x-6}-1)(\sqrt{x-6}+1)}{(7-x)(\sqrt{x-6}+1)}$

$= \frac{x-6-1}{(7-x)(\sqrt{x-6}+1)}$

$= \frac{\cancel{x-7}}{-\cancel{(x-7)}(\sqrt{x-6}+1)}$

$= \frac{-1}{(\sqrt{x-6}+1)}$

$\lim_{x\to 7}\left(\frac{(\sqrt{x-6}-1)}{7-x}\right) = -\lim_{x\to 7}\frac{1}{\sqrt{x-6}+1}$

$= -\frac{1}{\sqrt{1}+1}$

$= \frac{-1}{2}$

(b) $\lim_{\theta\to 0}\frac{\tan^2\theta}{\theta} = \lim_{\theta\to 0}\left(\frac{\sin^2\theta}{\theta}\right)\left(\frac{1}{\cos^2\theta}\right)$

$= \lim_{\theta\to 0}(\sin\theta)\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)\lim_{\theta\to 0}\left(\frac{1}{\cos^2\theta}\right)$

$= (0)(1)(1) = 0$

35 $\lim_{x\to 0}\left(\frac{5\sin x + \cos x - 1}{4x}\right)$

$= \lim_{x\to 0}\frac{5}{4}\left(\frac{\sin x}{x}\right) + \lim_{x\to 0}\left(\frac{\cos x - 1}{4x}\right)$

$= \frac{5}{4}\lim_{x\to 0}\left(\frac{\sin x}{x}\right) + \frac{1}{4}\lim_{x\to 0}\left(\frac{\cos x - 1}{x}\right)$

$= \frac{5}{4}$

36 $\lim_{x\to 0}\frac{\sin^2 x + \sin x(\cos x - 1)}{x^2}$

$= \lim_{x\to 0}\left[\frac{\sin^2 x}{x^2} + \left(\frac{\sin x}{x}\right)\left(\frac{\cos x - 1}{x}\right)\right]$

$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\lim_{x\to 0}\left(\frac{\sin x}{x}\right) + \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\lim_{x\to 0}\left(\frac{\cos x - 1}{x}\right)$

$= (1)(1) + (1)(0)$

$= 1$

Review exercise 12

1 $\lim_{x\to\infty}\frac{4x^5 + x^2 + x - 2}{4x^3 + 5} = \lim_{x\to\infty}\frac{4 + \frac{1}{x^3} + \frac{1}{x^4} - \frac{2}{x^5}}{\frac{4}{x^2} + \frac{5}{x^5}}$

$= \frac{4}{0} \to \infty$

2 $\lim_{x\to\infty} \frac{7x^3 - 2x^2 + x}{3x^2 - 1} = \lim_{x\to\infty} \frac{7 - \frac{2}{x} + \frac{1}{x^3}}{\frac{3}{x} - \frac{1}{x^3}}$

$= \frac{7}{0} \to \infty$

3 $\lim_{x\to -1} \frac{4x^2 - 3x + 2}{x^2 + x + 2} = \frac{4 + 3 + 2}{1 - 1 + 2} = \frac{9}{2}$

4 (a) $\lim_{x\to 2}\left(\frac{x^3 + x^2 + x - 14}{x^3 - x - 6}\right) = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2 + 3x + 7)}{\cancel{(x-2)}(x^2 + 2x + 3)}$

$= \lim_{x\to 2}\left(\frac{x^2 + 3x + 7}{x^2 + 2x + 3}\right)$

$= \frac{4 + 6 + 7}{4 + 4 + 3} = \frac{17}{11}$

(b) $\lim_{x\to \frac{-5}{4}} \frac{12x^2 + 23x + 10}{4x^2 + 13x + 10} = \lim_{x\to \frac{-5}{4}} \frac{\cancel{(4x+5)}(3x + 2)}{\cancel{(4x+5)}(x + 2)}$

$= \lim_{x\to \frac{-5}{4}}\left(\frac{3x + 2}{x + 2}\right) = \frac{3\left(\frac{-5}{4}\right) + 2}{\frac{-5}{4} + 2} = \frac{\frac{-7}{4}}{\frac{3}{4}} = \frac{-7}{3}$

5 $\lim_{\theta\to 0} \frac{\sin 2\theta}{\sin\theta} = \frac{\lim_{\theta\to 0} 2\left(\frac{\sin 2\theta}{2\theta}\right)}{\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)}$

$= \frac{2(1)}{1} = 2$

6 $\lim_{\theta\to 0} \frac{\cos 6\theta - 1}{2\theta} = 3\lim_{\theta\to 0} \frac{\cos 6\theta - 1}{6\theta}$

$= 3(0) = 0$

7 $\lim_{x\to 0} \frac{\sin 3x \sin 5x}{7x^2} = \lim_{x\to 0}\left(\frac{\sin 3x}{7x}\right)\left(\frac{\sin 5x}{x}\right)$

$= \frac{1}{7}\lim_{x\to 0}\left(\frac{3\sin 3x}{3x}\right)\lim_{x\to 0}\left(\frac{5\sin 5x}{5x}\right)$

$= \frac{1}{7}(3)(1)(5)(1) = \frac{15}{7}$

8 $\lim_{x\to -3} \frac{\sqrt{3 - 5x}}{x + 1} = \frac{\sqrt{3 + 15}}{-2} = \frac{\sqrt{18}}{-2} = \frac{-3\sqrt{2}}{2}$

9 (a) $\lim_{x\to \frac{-1}{2}} \frac{2x^2 + 5x + 2}{2x^2 + 9x + 4} = \lim_{x\to -\frac{1}{2}} \frac{\cancel{(2x+1)}(x + 2)}{\cancel{(2x+1)}(x + 4)}$

$$= \lim_{x \to \frac{-1}{2}} \frac{(x+2)}{(x+4)} = \frac{2-\frac{1}{2}}{4-\frac{1}{2}} = \frac{\frac{3}{2}}{7/2} = \frac{3}{7}$$

(b) $x^2 + 3x + 2 = 0$

$(x+1)(x+2) = 0$

$x = -1, -2$

f(x) is continuous everywhere except at x = –2, –3

10 (a) $\lim_{\theta \to 0} \frac{\sin 5\theta}{\theta} = \lim_{\theta \to 0} \frac{5\sin 5\theta}{5\theta} = 5(1) = 5$

(b) $|2x-3| - 5 = 0$

$|2x-3| = 5$

2x–3 = 5, 2x–3 = –5

x = 4, x= –1

The function is continuous everywhere except at x = 4, x = –1

11 $\lim_{\theta \to 0}\left(\frac{\tan 5\theta}{\theta}\right) = \lim_{\theta \to 0}\left(\frac{\sin 5\theta}{\theta \cos 5\theta}\right)$

$= \lim_{\theta \to 0} 5\left(\frac{\sin 5\theta}{5\theta}\right) \lim_{\theta \to 0} \frac{1}{\cos 5\theta}$

$= 5(1)(1) = 5$

12 $\lim_{x \to 0} \frac{\tan 2x - 4x}{\sin 3x - 7x} = \lim_{x \to 0} \frac{\frac{\sin 2x}{\cos 2x} - 4x}{\sin 3x - 7x}$

$= \lim_{x \to 0} \frac{\frac{\sin 2x}{x\cos 2x} - \frac{4x}{x}}{\frac{\sin 3x}{x} - \frac{7x}{x}}$

$= \frac{\lim_{x \to 0}\left(\frac{2\sin 2x}{2x}\right) \lim_{x \to 0}\left(\frac{1}{\cos 2x}\right) - \lim_{x \to 0} 4}{\lim_{x \to 0}\left(\frac{3\sin 3x}{3x}\right) - \lim_{x \to 0} 7}$

$= \frac{(2)(1) - 4}{3(1) - 7} = \frac{-2}{-4} = \frac{1}{2}$

13 (a) $4x^2 - 11x - 3 = 0$

(4x + 1)(x – 3) = 0

$x = -\frac{1}{4}, 3$

f(x) is continuous everywhere except at $x = -\frac{1}{4}, 3$

(b) $\lim_{x \to 0} \frac{\sin 4x}{x} = \lim_{x \to 0} 4\left(\frac{\sin 4x}{4x}\right) = 4(1) = 4$

$\lim_{x \to 0} \frac{\sin 4x}{\sin 5x} = \lim_{x \to 0}\left(\frac{\frac{\sin 4x}{x}}{\frac{\sin 5x}{x}}\right) = \frac{\lim_{x \to 0}\left(\frac{4\sin 4x}{4x}\right)}{\lim_{x \to 0}\left(\frac{5\sin 5x}{5x}\right)}$

$= \frac{4}{5}$

14 (a) $\frac{\sqrt{x+4}-3}{x-5} = \frac{\sqrt{x+4}-3}{x-5} \times \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$

$= \frac{(x+4)-9}{(x-5)(\sqrt{x+4}+3)}$

$= \frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x+4}+3)}$

$= \frac{1}{\sqrt{x+4}+3}$

$\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5} = \lim_{x\to 5} \frac{1}{\sqrt{x+4}+3}$

$= \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

(b) $\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$

$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = \lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{x}\right)$

$= 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$

$= 2(1)\,(1) = 2$

(c) $|3x - 1| - 8 = 0$

$|3x - 1| = 8$

$3x - 1 = 8,\ 3x - 1 = -8$

$x = 3,\ 3x = -7$

$x = -7/3$

the function is continuous everywhere except at $x = 3,\ x = \frac{-7}{3}$

15 $f(x) = \frac{9x^2 - 3x - 2}{3x^2 + 13x + 4}$

$= \frac{(3x+1)(3x-2)}{(3x+1)(x+4)}$

$\therefore$ Discontinuous at $x = \frac{-1}{3},\ x = -4$

At $x = \frac{-1}{3}$, point discontinuity which is removable

x = 4, infinite discontinuity which is non-removable